Radiative Processes in Astrophysics

Lecture 5
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Thomson Scattering (Electron Scattering)

- Recall the dipole formula $$\frac{dP}{d\Omega} = \frac{dW}{dtd\Omega} = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3}\sin^2\Theta, \quad P = \frac{2\ddot{\mathbf{d}}^2}{3c^3}$$
- Let us consider the **process in which a free charged particle (electron) radiates in response to an incident electromagnetic wave**.
- In non-relativistic case, we may neglect magnetic force.

  magnetic/electric force ratio in Lorentz force: $F_B/F_E \sim (v/c)B/E = v/c \ll 1$
- Consider a monochromatic wave with frequency ω_0 and linearly polarized in direction $\hat{\boldsymbol{\epsilon}}$:

$$\mathbf{E} = \hat{\boldsymbol{\epsilon}} E_0 \sin\omega_0 t$$

Thus the force on a particle with the charge *e* is

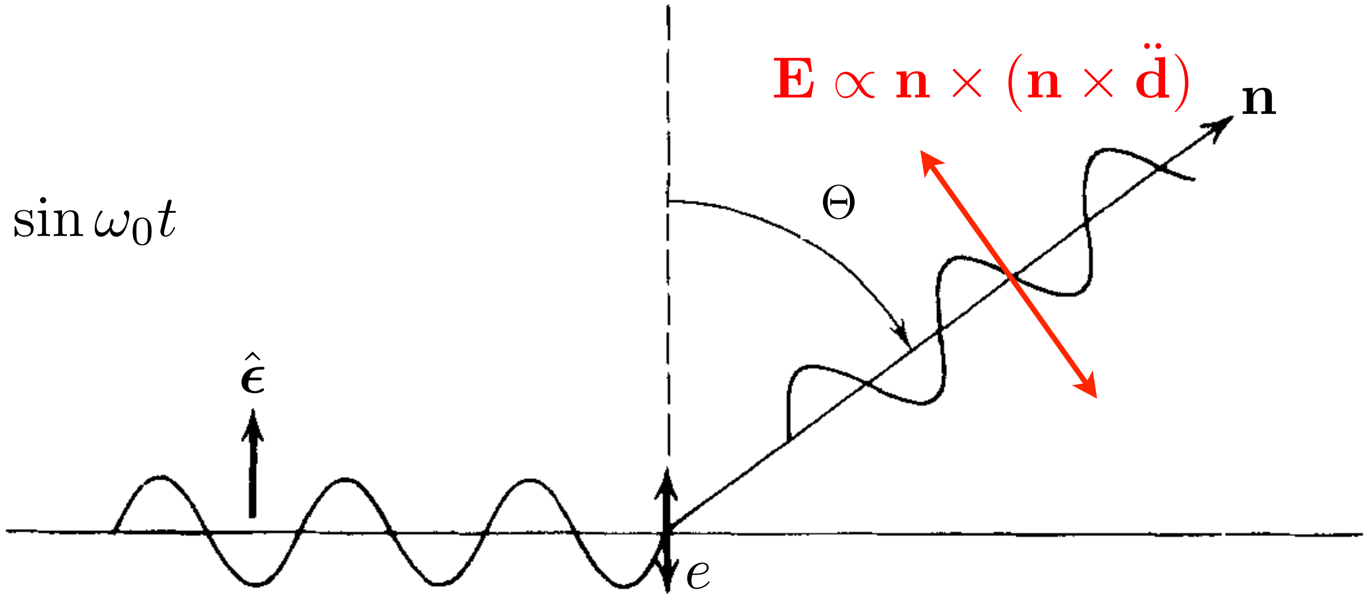
$$\mathbf{F} = e\mathbf{E} = \hat{\boldsymbol{\epsilon}} e E_0 \sin\omega_0 t$$

the acceleration of the electron is

$$\ddot{\mathbf{r}} = \hat{\boldsymbol{\epsilon}} \frac{eE_0}{m}\sin\omega_0 t, \quad \ddot{\mathbf{d}} = e\ddot{\mathbf{r}} = \hat{\boldsymbol{\epsilon}} \frac{e^2E_0}{m}\sin\omega_0 t$$

the dipole moment is

$$\mathbf{d} = -\hat{\boldsymbol{\epsilon}} \left(\frac{e^2E_0}{m\omega_0^2}\right)\sin\omega_0 t$$



- We obtain the time-averaged power per solid angle $\left(\langle \sin^2 \omega_0 t \rangle = 1/2\right)$:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{< \ddot{\mathbf{d}}^2 >}{4\pi c^3} \sin^2 \Theta = \frac{e^4 E_0^2}{8\pi m^2 c^3} \sin^2 \Theta, \quad \langle P \rangle = \frac{e^4 E_0^2}{3m^2 c^3}$$

Note that the time-averaged incident flux is

$$\langle S \rangle = \frac{c}{8\pi} E_0^2$$

The **differential cross section**, $\frac{d\sigma}{d\Omega}$, for linearly polarized radiation is obtained by

$$\frac{d\sigma}{d\Omega} \equiv \left\langle \frac{dP}{d\Omega} \right\rangle / \langle S \rangle\,, \quad \boxed{\therefore \frac{d\sigma}{d\Omega} = \frac{e^4}{m^2 c^4} \sin^2 \Theta = r_0^2 \sin^2 \Theta, \quad r_0 \equiv \frac{e^2}{mc^2}}$$

where the quantity r_0 gives a measure of the “size” of the point charge. (Note electrostatic potential energy $e\phi = e^2/r_0$).

For an electron, the classical electron radius has a value $r_0 = 2.82 \times 10^{-13}$ cm.

The total cross section is found by integrating over solid angle.

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi r_0^2 \int_{-1}^{1} (1 - \mu^2) d\mu = \frac{8\pi}{3} r_0^2$$

For an electron, the scattering process is then called Thomson scattering or electron scattering, and the **Thomson cross section** is

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3} r_0^2 = 6.652 \times 10^{-25} \text{ cm}^2$$

- Note:

  The total and differential cross sections are frequency independent.

  The scattered radiation is linearly polarized in the plane of the incident polarization vector $\hat{\epsilon}$ and the direction of scattering $\mathbf{n}$.

  $\sigma \propto 1/m^2$: electron scattering is larger than ions by a factor of $(m_{\rm p}/m_e)^2 = (1836)^2 \approx 3.4 \times 10^6$.

  We have implicitly assumed that electron recoil is negligible. This is only valid for nonrelativistic energies. For higher energies, the (quantum-mechanical) Klein-Nishina cross section has to be used.

- What is the cross section for scattering of **unpolarized radiation**?

  An unpolarized beam can be regarded as the independent superposition of two linear-polarized beams with perpendicular axes.

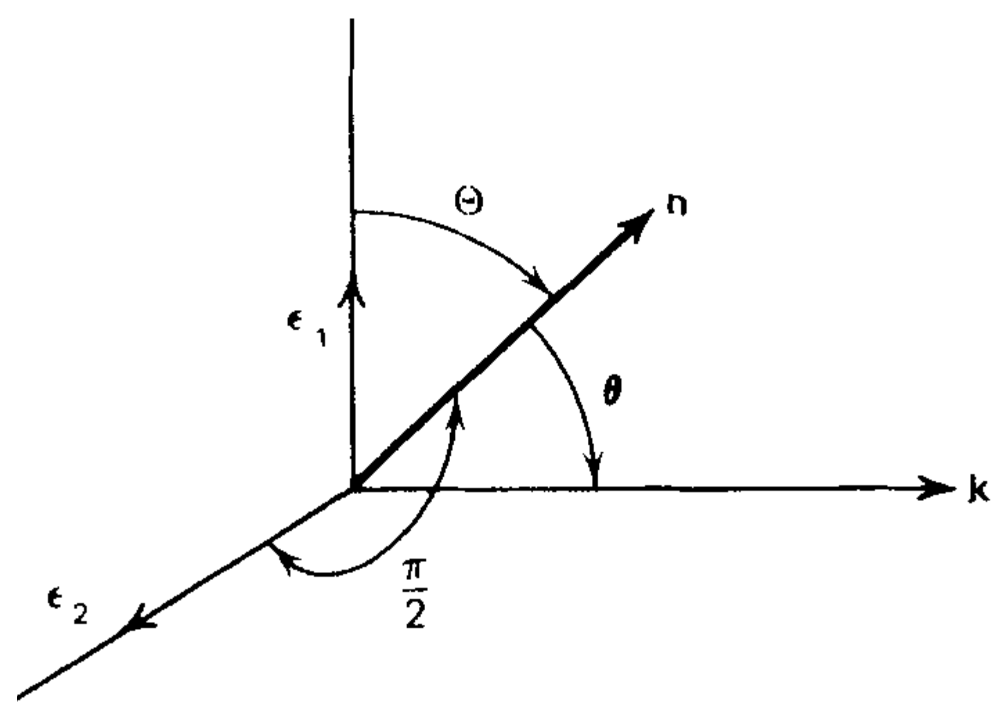
  Let us assume that $\mathbf{n}$ = direction of scattered radiation

  $\mathbf{k}$ = direction of incident radiation

  Choose

  the first electric field along $\hat{\epsilon}_1$, which is in the $\mathbf{n} - \mathbf{k}$ plane

  the second one along $\hat{\epsilon}_2$ orthogonal to this plane and to $\mathbf{n}$



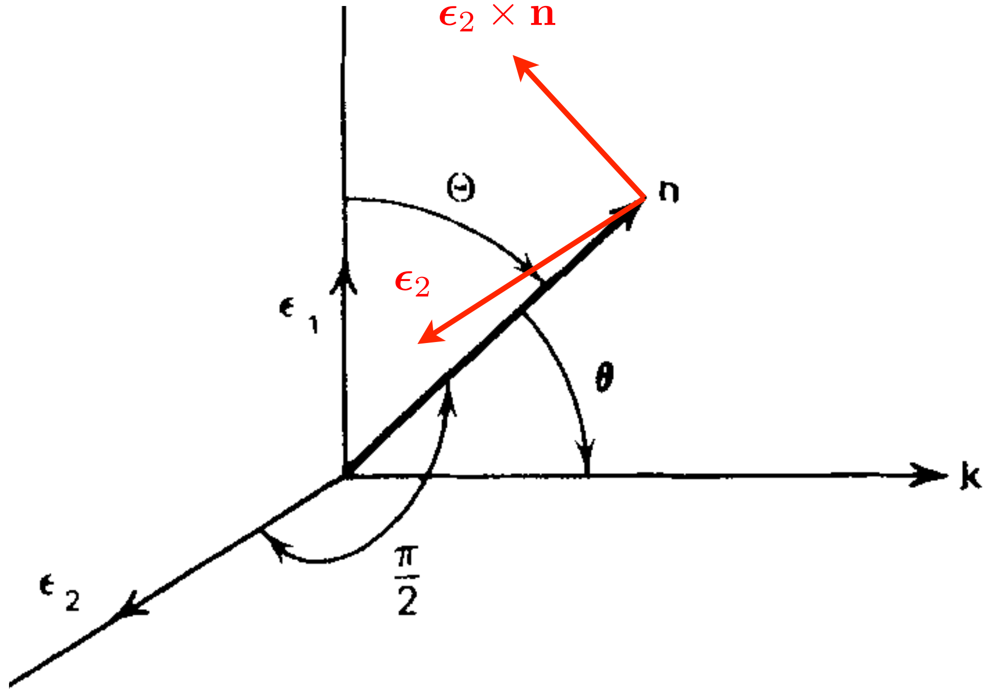
- Let Θ = angle between $\boldsymbol{\epsilon}_1$ and $\mathbf{n}$, and note that angle between $\boldsymbol{\epsilon}_2$ and $\mathbf{n}$ = $\pi/2$.

  $\theta = \pi/2 - \Theta$ = angle between the scattered wave and incident wave

  Then, the differential cross section for unpolarized radiation

  is the average of the cross sections for scattering of two electric fields.

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left[\left(\frac{d\sigma}{d\Omega}\right)_{\epsilon_2} + \left(\frac{d\sigma}{d\Omega}\right)_{\epsilon_1}\right]$$
$$= \frac{1}{2}\left[\left(\frac{d\sigma(\pi/2)}{d\Omega}\right)_{\text{pol}} + \left(\frac{d\sigma(\Theta)}{d\Omega}\right)_{\text{pol}}\right]$$
$$= \frac{1}{2} r_0^2 \left(1 + \sin^2\Theta\right)$$
$$= \frac{1}{2} r_0^2 \left(1 + \cos^2\theta\right)$$



This depends only on the angle between the incident and scattered directions, as it should for unpolarized radiation.

Total cross section: $$\sigma_{\text{unpol}} = \int \left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} d\Omega = \pi r_0^2 \int_{-1}^{1} (1+\mu^2)\,d\mu$$
$$= \frac{8\pi}{3} r_0^2$$
$$= \sigma_{\text{pol}}$$

Properties of Thomson Scattering

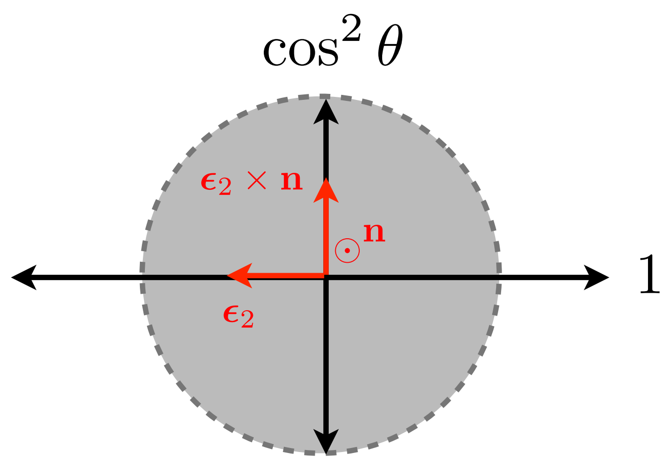
- Forward-backward symmetry: differential cross section is symmetric under $\theta \to -\theta$.
- Total cross section of unpolarized incident radiation = total cross section for polarized incident radiation. This is because the electron at rest has no preferred direction defined.
- **Scattering creates polarization**

The scattered intensity is proportional to $1+\cos^2\theta$, of which 1 arises from the incident electric field along $\boldsymbol{\epsilon}_2$ and $\cos^2\theta$ from the incident electric field along $\boldsymbol{\epsilon}_1$.

"$\cos^2\theta$" of the polarization along $\boldsymbol{\epsilon}_2$ will be cancelled out by the independent polarization along $\boldsymbol{\epsilon}_2 \times \mathbf{n}$.

Therefore, the degree of polarization of the scattered wave:

$$\Pi = \frac{1-\cos^2\theta}{1+\cos^2\theta}$$



Electron scattering of a completely unpolarized incident wave produces a scattered wave with some degree of polarization.

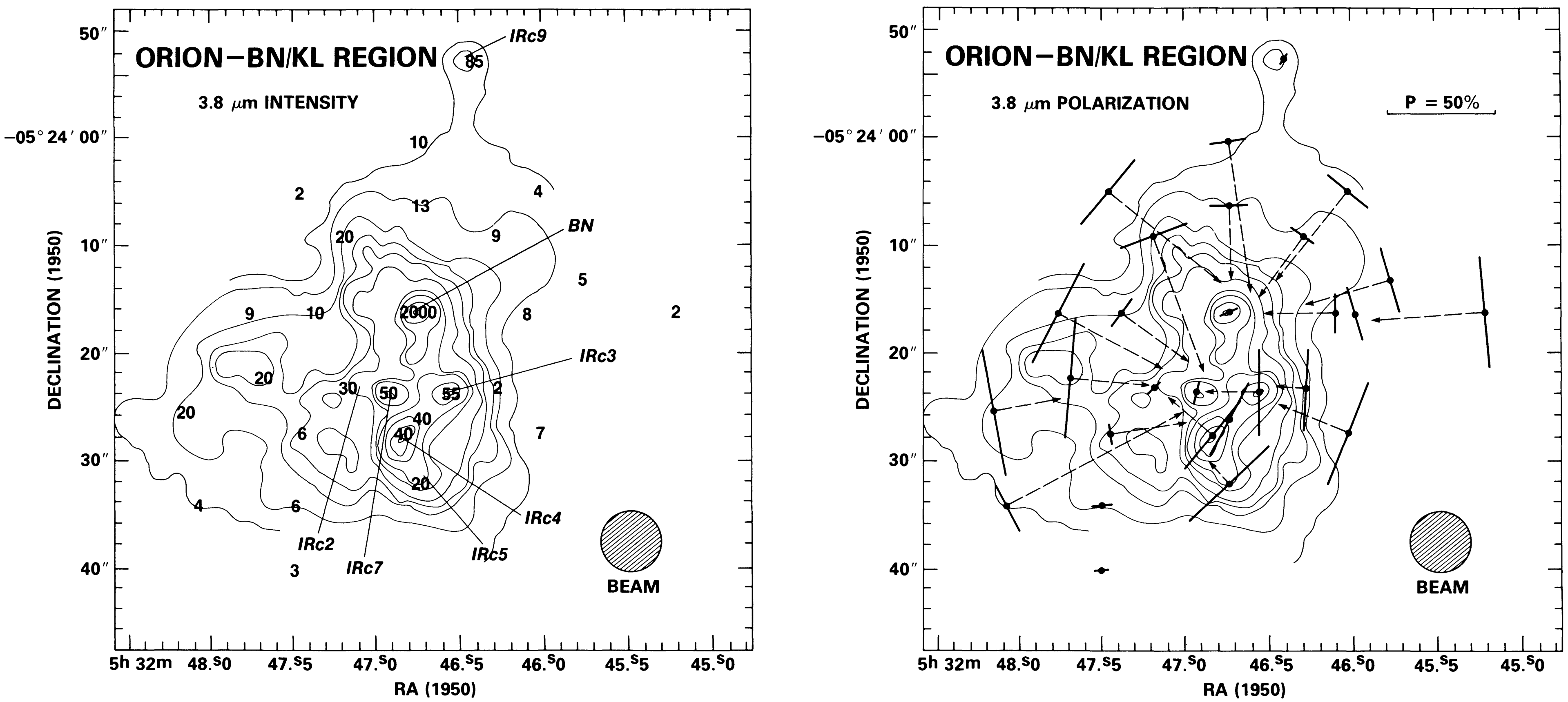
<u>No net polarization along the incident direction</u> $(\theta = 0)$, since, by symmetry, all directions are equivalent.

<u>100% polarization perpendicular to the incident direction</u> $(\theta = \pi/2)$, since the electron's motion is confined to a plane normal to the incident direction.

Astrophysical Applications of Polarization by Scattering

- Detection of a concentric pattern of polarization vectors in an extended region indicates that the light comes via scattering from a central point source.

Werner et al. (1983, ApJL, 265, L13)



- Left map shows the IR intensity map at 3.8 um of the Becklin-Neugebauer/Kleinmann-Low region of Orion. It is not easy to identify which bright spots correspond to locations of possible protostars.
- However, the polarization map singles out only two positions of intrinsic luminosity: IRc2 (now known to be an intense protostellar wind) and BN (suspected to be a relatively high-mass star)
- All the other bright spots (IRc3 through 7) correspond to IR reflection nebulae.

Radiation from Harmonically Bound Particles

- **Thomson Model of an Atom**: Electrons have equilibrium positions in an atom like raisins in a raisin pudding. When perturbed slightly away from this state, they will vibrate about their equilibrium positions like a harmonic oscillator, with a characteristic frequency.
- **Undriven Harmonically Bound Particles** (free oscillator)

The electron oscillation in a Thomson atom can be viewed as a classical oscillating dipole. Since an oscillating electron represents a continuously accelerating charge, the electron will radiate energy. Then, the radiative loss rate of energy, averaged over one cycle of the oscillating dipole will be

$$\frac{dW}{dt} = -\frac{2e^2 \left\langle |\ddot{\mathbf{x}}|^2 \right\rangle}{3c^3}$$

where

$$\left\langle |\ddot{\mathbf{x}}|^2 \right\rangle \equiv \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} dt$$

$$= \frac{1}{\tau} \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \Big|_{-\tau/2}^{\tau/2} - \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} dt$$

The period τ and frequency ω_0 of the oscillator is related by $\tau \equiv 2\pi/\omega_0$.

Here, we note that $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are 90° out of phase. Then,

$$\left\langle |\ddot{\mathbf{x}}|^2 \right\rangle = -\frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} dt = -\left\langle \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \right\rangle \quad \rightarrow \quad \frac{dW}{dt} = \frac{2e^2 \left\langle \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \right\rangle}{3c^3}$$

- Abraham-Lorentz formula: We can identify the radiation reaction force (the damping of a charge's motion which arises because of the emission of radiation) by noting that

$$\frac{dW}{dt} = \langle \mathbf{F}_{\rm rad} \cdot \dot{\mathbf{x}} \rangle \quad \rightarrow \quad \mathbf{F}_{\rm rad} \equiv \frac{2e^2}{3c^3}\dddot{\mathbf{x}} \quad : \qquad \text{Abraham-Lorentz formula}$$

This formula depends on the derivative of acceleration. This increases the degree of the equation of motion of a particle and can lead to some nonphysical behavior if not used properly and consistently.

For a simple harmonic oscillator with a frequency ω_0, we can avoid the difficulty by using

$$\dddot{\mathbf{x}} = -\omega_0^2 \dot{\mathbf{x}}$$

This is a good assumption as long as the energy is to be radiated on a time scale that is long compared to the period of oscillation. In this regime, radiation reaction may be considered as a *perturbation* on the particle's motion. We then rewrite the radiation reaction force as

$$\mathbf{F}_{\rm rad} = -\frac{2e^2\omega_0^2}{3c^3}\dot{\mathbf{x}} = -m\gamma\dot{\mathbf{x}}, \qquad \gamma \equiv \frac{2e^2\omega_0^2}{3mc^3} \quad : \qquad \text{damping constant}$$

Note $\gamma = A_{21}$

(a) condition for the approximation:

$$\gamma/\omega_0 = (2e^2/3mc^2)(\omega_0/c) = (2/3)(r_e/\lambda_0)2\pi \ll 1 \quad \text{for } \lambda_0 \gg r_e = 2.82 \times 10^{-13} \text{ cm}$$

In this limit, radiation damping has a well-defined notion.

(b) condition for the approximation:

T = the time interval over which the kinetic energy of the particle is changed substantially by the emission of radiation:

$$T \sim \frac{mv^2}{dW/dt} \sim \frac{3mc^3}{2e^2}\left(\frac{v}{a}\right)^2$$

t_p = the typical orbital time scale for the particle: $t_p \sim \frac{v}{a}$ or $t_p = \frac{2\pi}{\omega_0}$

Then, the condition becomes

$$\frac{T}{t_p} \gg 1 \quad \rightarrow \quad \frac{3mc^3}{2e^2} t_p = \frac{t_p}{\tau_c} \gg 1$$

where $\tau_c \equiv \frac{2}{3}\frac{e^2}{mc^3} = \frac{2}{3}\frac{r_e}{c}$ $(\sim 10^{-23}\ \mathrm{s})$

is the time for radiation to cross a distance comparable to the classical electron radius.

In terms of frequency of the oscillator, this condition is equivalent to:

$$\frac{2\pi}{\tau_c} = 3\pi\frac{c}{r_e} \equiv \omega_c \gg \omega_0$$

In terms of wavelength of the oscillator,

$$\lambda_0 = \frac{2\pi c}{\omega_0} \gg \lambda_c \equiv \frac{2\pi c}{\omega_c} = \frac{2}{3} r_e \ (\sim\ 2\times 10^{-13}\ \mathrm{cm} = 2\times 10^{-5}\text{Å})$$

Therefore, in most cases, the approximation is valid.

- Equation of motion of the electron in a Thomson atom, including the radiation damping force, is

$$\ddot{\mathbf{x}} + \gamma\dot{\mathbf{x}} + \omega_0^2\mathbf{x} = 0$$

This equation may be solved by assuming that $x(t) \propto e^{\alpha t}$.

$$\alpha^2 + \gamma\alpha + \omega_0^2 = 0 \quad \rightarrow \quad \alpha = -(\gamma/2) \pm \sqrt{(\gamma/2)^2 - \omega_0^2}$$
$$= -\gamma/2 \pm i\omega_0 t + \mathcal{O}(\gamma^2/\omega_0^2)$$

Assuming initial conditions

$$x(0) = x_0,\ \dot{x}(0) = 0 \text{ at } t = 0$$

we have

$$x(t) = \frac{1}{2}x_0\left[e^{-(\gamma/2 - i\omega_0)t} + e^{-(\gamma/2 + i\omega_0)t}\right]$$

- Power spectrum:

$$\bar{x}(\omega) = \frac{1}{2\pi}\int_0^\infty x(t)e^{i\omega t}dt = \frac{x_0}{4\pi}\left[\frac{1}{\gamma/2 - i(\omega + \omega_0)} + \frac{1}{\gamma/2 - i(\omega - \omega_0)}\right]$$

This becomes large in the vicinity of $\omega = \omega_0$ and $\omega = -\omega_0$.

We are ultimately interested only in positive frequencies, and only in regions in which the values become large. Therefore, we obtain

$$\bar{x}(\omega) \approx \frac{x_0}{4\pi}\frac{1}{\gamma/2 - i(\omega - \omega_0)}, \quad |\bar{x}(\omega)|^2 = \left(\frac{x_0}{4\pi}\right)^2 \frac{1}{(\omega - \omega_0)^2 + (\gamma/2)^2}$$

Recall $$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} e^2 |\bar{x}(\omega)|^2$$

Energy radiated per unit frequency:

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} \frac{e^2 x_0^2}{(4\pi)^2} \frac{1}{(\omega-\omega_0)^2 + (\gamma/2)^2} = \frac{1}{2} m \left(\frac{\omega^4}{\omega_0^2}\right) x_0^2 \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$
$$\approx \frac{1}{2} m \omega_0^2 x_0^2 \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

For a harmonic oscillator, note that the equation of motion is $\mathbf{F} = -k\mathbf{x} = -m\omega_0^2\mathbf{x}$, spring constant is $k = m\omega_0^2$, and the potential energy (energy stored in spring) is $(1/2)kx_0^2$.

From $$\int_{-\infty}^{\infty} \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2} d\omega = \frac{1}{\pi} \tan^{-1}\{2(\omega-\omega_0)/\gamma\}|_{-\infty}^{\infty} = 1$$

Total emitted energy = initial potential energy of the oscillator:

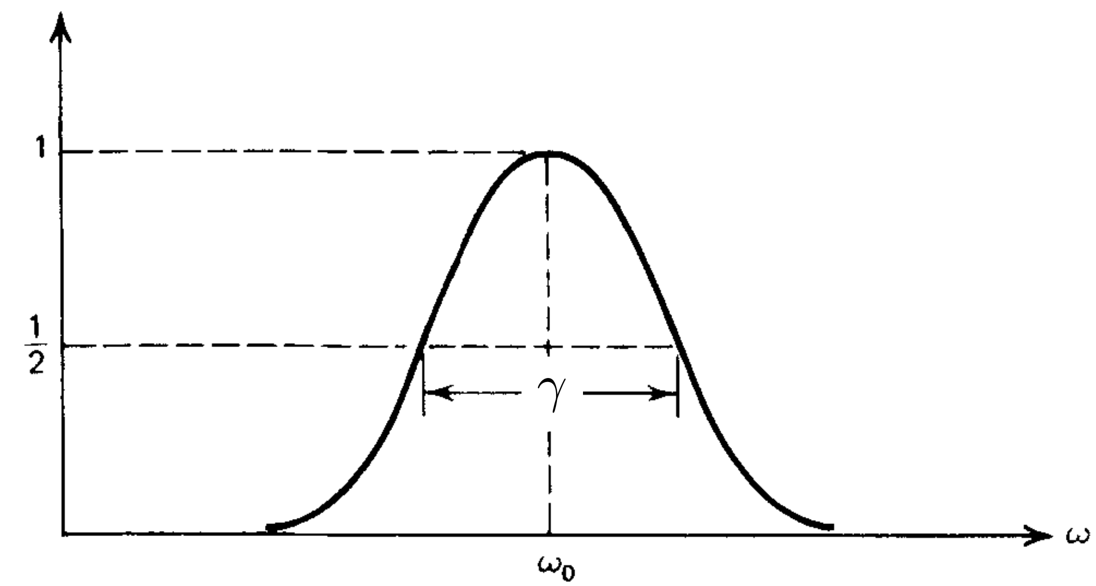
$$W = \int_0^{\infty} \frac{dW}{d\omega} d\omega = \frac{1}{2} k\omega_0^2$$

Profile of the emitted spectrum: $$\phi(\omega) = \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$ **Lorentz (natural) profile**

Damping constant is the full width at half maximum (FWHM).

$$\phi(\omega) = \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$\phi(\nu) = \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$$



Note $\phi(\omega)d\omega = \phi(\nu)d\nu$

The line width $\Delta\omega = \gamma$ is a universal constant when expressed in terms of wavelength:

$$\lambda = \frac{2\pi c}{\omega}$$

$$\Delta\lambda = 2\pi c \frac{\Delta\omega}{\omega^2} = 2\pi c \frac{2}{3}\frac{r_e}{c}$$

$$= \frac{4}{3}\pi r_e$$

$$= 1.2 \times 10^{-4} \text{Å}$$

- **Driven Harmonically Bound Particles** (forced oscillators)

Electron's equation of motion

$$\ddot{\mathbf{x}} + \gamma\dot{\mathbf{x}} + \omega_0^2\mathbf{x} = -\frac{e\mathbf{E}_0}{m}e^{i\omega t}$$

Rybicki & Lightman use the following equation.

$$\ddot{\mathbf{x}} - (\gamma/\omega_0^2)\dddot{\mathbf{x}} + \omega_0^2\mathbf{x} = -\frac{e\mathbf{E}_0}{m}e^{i\omega t}$$

Steady-state solution of this equation:

$$\mathbf{x} = \mathbf{x}_0 e^{i\omega t} \equiv |\mathbf{x}_0| e^{i(\omega t+\delta)} \quad \rightarrow \quad (-\omega^2 + i\omega\gamma + \omega_0^2)\mathbf{x}_0 e^{i\omega t} = -\frac{e\mathbf{E}_0}{m}e^{i\omega t}$$

$$\mathbf{x}_0 = \frac{(e/m)\mathbf{E}_0}{(\omega^2 - \omega_0^2) - i\omega\gamma}$$

$$\mathbf{x}_0 = |\mathbf{x}_0|e^{i\delta} \propto (\omega^2 - \omega_0^2) + i\omega\gamma \quad \rightarrow \quad \delta = \tan^{-1}\left(\frac{\omega\gamma}{\omega^2 - \omega_0^2}\right)$$

The response is slightly out of phase with respect to the imposed field.

For $\omega > \omega_0$, the particle "leads" the driving force and for $\omega < \omega_0$ it "lags."

Time-averaged total power radiated:

$$P = \left\langle \frac{dW}{dt} \right\rangle = \frac{2e^2 \left\langle |\ddot{\mathbf{x}}|^2 \right\rangle}{3c^3} = \frac{e^2\omega^4|\mathbf{x}_0|^2}{3c^3}$$

$$= \frac{e^4 E_0^2}{3m^2c^3} \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + (\omega\gamma)^2}$$

- Scattering cross section:

$$\sigma_{\rm sca} \equiv \frac{\langle P \rangle}{\langle S \rangle}, \quad \langle S \rangle = \frac{c}{8\pi} E_0^2 \quad \longrightarrow \quad \sigma_{\rm sca}(\omega) = \frac{8\pi e^4}{3m^2c^4} \frac{\omega^4}{(\omega^2-\omega_0^2)^2 + (\omega\gamma)^2}$$

$$= \sigma_T \frac{\omega^4}{(\omega^2-\omega_0^2)^2 + (\omega\gamma)^2}$$

- Some Limiting Cases of Interest

(a) $\omega \gg \omega_0$ (Thomson scattering by free electron)

$$\sigma_{\rm sca} = \sigma_T = \frac{8\pi}{3} r_e^2$$

At high incident energies, the binding becomes negligible.

(b) $\omega \ll \omega_0$ (Rayleigh scattering by bound electron)

$$\boxed{\sigma_{\rm sca} = \sigma_T \left(\frac{\omega}{\omega_0} \right)^4}$$

The electric filed appears nearly static and produces a nearly static force.

Blue color of the sky at sunrise:

Red color of the sun at sunset: when the path through the atmosphere is longer, the blue and green components are removed almost completely leaving the longer wavelength orange and red

(c) $\omega \approx \omega_0$ (Resonance scattering of line radiation)

$$\sigma_{\rm sca}(\omega) \approx \sigma_T \frac{\omega_0^4}{(\omega-\omega_0)^2(2\omega_0)^2 + (\omega_0\gamma)^2}$$
$$= \sigma_T \frac{\omega_0^2/4}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$\sigma_T \frac{\omega_0^2}{4} = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \times \frac{1}{4} \times \left(\gamma \frac{3}{2}\frac{mc^3}{e^2\omega_0^2}\right) = 2\pi^2 \frac{e^2}{mc}(\gamma/2\pi) \longrightarrow$$

Note $\sigma_{\rm scat}(\omega) = \sigma_\nu(\nu)$

$$\sigma_{\rm sca}(\omega) = \frac{2\pi^2 e^2}{mc}\frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$
$$\sigma_{\rm sca}(\nu) = \frac{\pi e^2}{mc}\frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$$

In the neighborhood of the resonance, the shape of the scattering cross section is the same as the emission from the free oscillator.

Total scattering cross section: $\int_0^\infty \sigma(\omega)d\omega = \frac{2\pi^2 e^2}{mc}, \quad \int_0^\infty \sigma(\nu)d\nu = \frac{\pi e^2}{mc}$

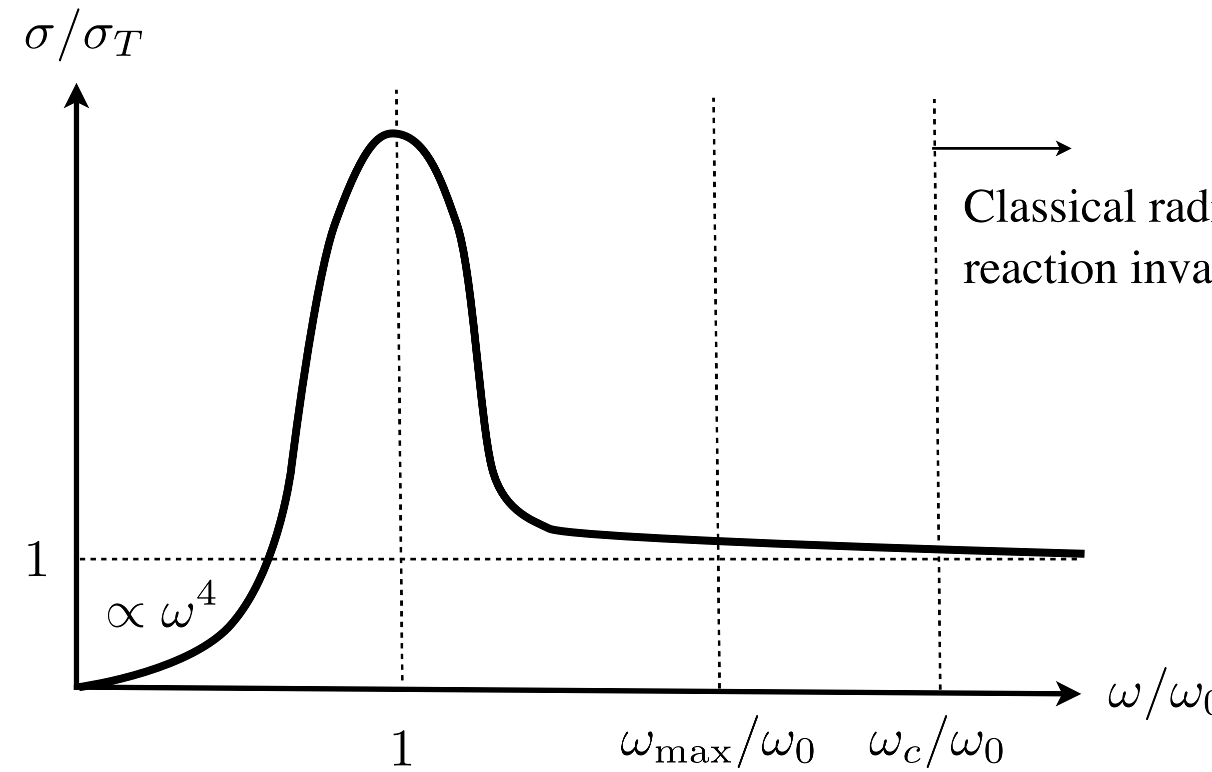
In evaluating this integral, we have apparently neglected a divergence, since the cross section approaches σ_T for large ω .

However, note that the approximate formula for radiation reaction is only valid for $\omega_0 \ll \omega_c$. Therefore, we must cut off the integral at a $\omega_{\rm max}$ such that $\omega_0 \ll \omega_{\rm max} \ll \omega_c$.

We also note that the contribution to the integral from the constant Thomson limit is less than

$$\int_0^{\omega_{\max}} \sigma_T d\omega = \sigma_T \omega_{\max} \ll \sigma_T \omega_c = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \times 3\pi\left(\frac{mc^3}{e^2}\right) = \frac{8\pi^2 e^2}{mc} \approx \int_0^{\infty} \sigma_{\mathrm{sca}}(\omega) d\omega$$

The contribution is therefore negligible.



In the quantum theory of spectral lines, we obtain similar formulas, which are conveniently stated in terms of the classical results as

$$\int_0^{\infty} \sigma(\nu) d\nu = \frac{\pi e^2}{mc} f_{nn'}$$

where $f_{nn'}$ is called the *oscillator strength* or *f-value* for the transition between states *n* and *n'*.

Resonance Lines

Draine, Physics of the interstellar and intergalactic medium

Table 9.4 Selected Resonance Lines[a] with $\lambda < 3000$ Å

	Configurations	ℓ	u	$E_\ell/hc(\mathrm{cm}^{-1})$	λ_{vac}(Å)	$f_{\ell u}$
C IV	$1s^22s - 1s^22p$	$^2S_{1/2}$	$^2P^o_{1/2}$	0	1550.772	0.0962
		$^2S_{1/2}$	$^2P^o_{3/2}$	0	1548.202	0.190
N V	$1s^22s - 1s^22p$	$^2S_{1/2}$	$^2P^o_{1/2}$	0	1242.804	0.0780
		$^2S_{1/2}$	$^2P^o_{3/2}$	0	1242.821	0.156
O VI	$1s^22s - 1s^22p$	$^2S_{1/2}$	$^2P^o_{1/2}$	0	1037.613	0.066
		$^2S_{1/2}$	$^2P^o_{3/2}$	0	1037.921	0.133
C III	$2s^2 - 2s2p$	$^1S_0$	$^1P^o_1$	0	977.02	0.7586
C II	$2s^22p - 2s2p^2$	$^2P^o_{1/2}$	$^2D^o_{3/2}$	0	1334.532	0.127
		$^2P^o_{3/2}$	$^2D^o_{5/2}$	63.42	1335.708	0.114
N III	$2s^22p - 2s2p^2$	$^2P^o_{1/2}$	$^2D^o_{3/2}$	0	989.790	0.123
		$^2P^o_{3/2}$	$^2D^o_{5/2}$	174.4	991.577	0.110
C I	$2s^22p^2 - 2s^22p3s$	$^3P_0$	$^3P^o_1$	0	1656.928	0.140
		$^3P_1$	$^3P^o_2$	16.40	1656.267	0.0588
		$^3P_2$	$^3P^o_2$	43.40	1657.008	0.104
N II	$2s^22p^2 - 2s2p^3$	$^3P_0$	$^3D^o_1$	0	1083.990	0.115
		$^3P_1$	$^3D^o_2$	48.7	1084.580	0.0861
		$^3P_2$	$^3D^o_3$	130.8	1085.701	0.0957
N I	$2s^22p^3 - 2s^22p^23s$	$^4S^o_{3/2}$	$^4P_{5/2}$	0	1199.550	0.130
		$^4S^o_{3/2}$	$^4P_{3/2}$	0	1200.223	0.0862
O I	$2s^22p^4 - 2s^22p^33s$	$^3P_2$	$^3S^o_1$	0	1302.168	0.0520
		$^3P_1$	$^3S^o_1$	158.265	1304.858	0.0518
		$^3P_0$	$^3S^o_1$	226.977	1306.029	0.0519
Mg II	$2p^63s - 2p^63p$	$^2S_{1/2}$	$^2P^o_{1/2}$	0	2803.531	0.303
		$^2S_{1/2}$	$^2P^o_{3/2}$	0	2796.352	0.608
Al III	$2p^63s - 2p^63p$	$^2S_{1/2}$	$^2P^o_{1/2}$	0	1862.790	0.277
		$^2S_{1/2}$	$^2P^o_{3/2}$	0	1854.716	0.557

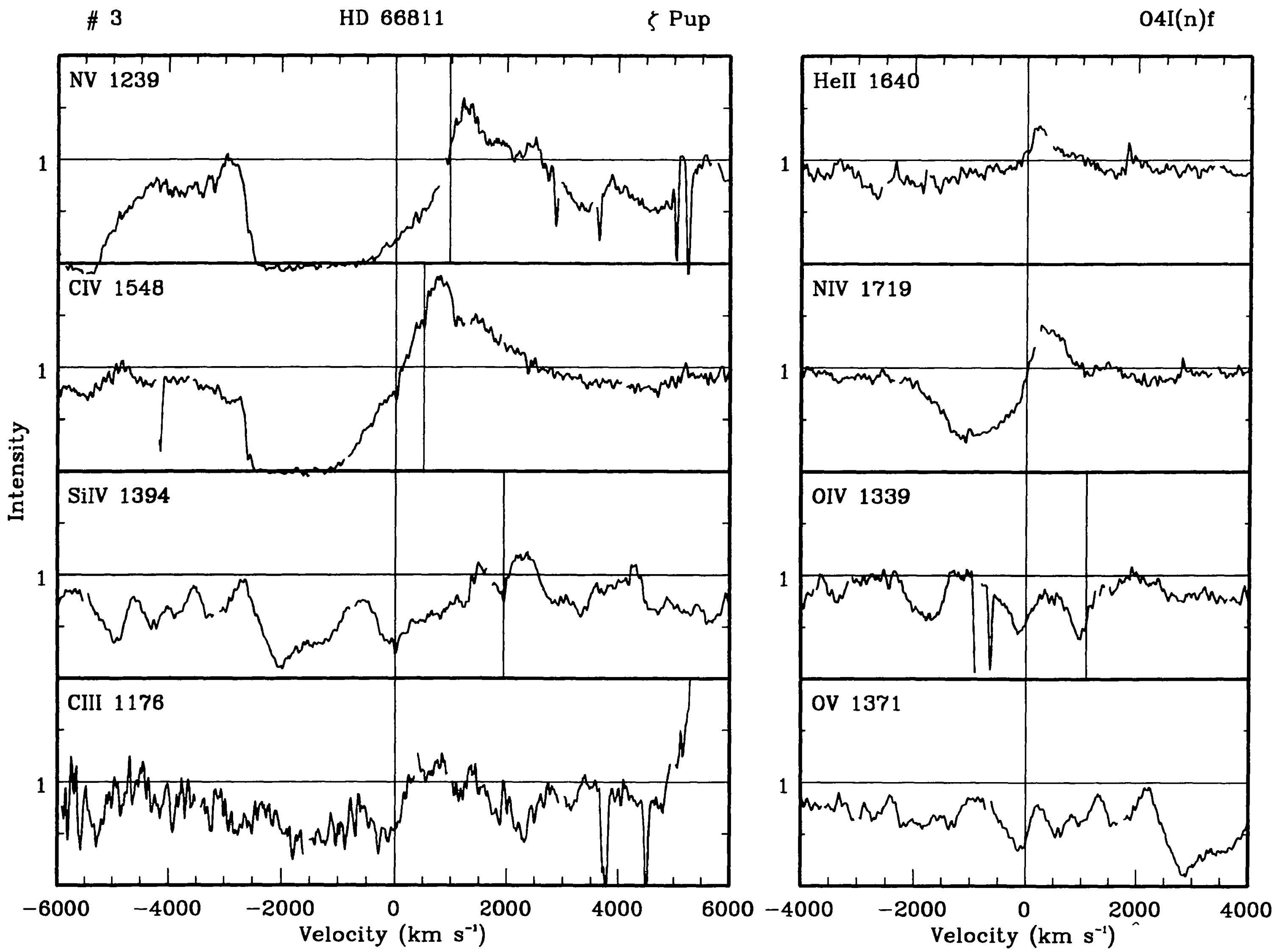
Table 9.4 contd.

	Configurations	ℓ	u	$E_\ell/hc\,(\mathrm{cm}^{-1})$	λ_{vac}(Å)	$f_{\ell u}$
Mg I	$2p^63s^2 - 2p^63s3p$	$^1S_0$	$^1P^o_1$	0	2852.964	1.80
Al II	$2p^63s^2 - 2p^63s3p$	$^1S_0$	$^1P^o_1$	0	1670.787	1.83
Si III	$2p^63s^2 - 2p^63s3p$	$^1S_0$	$^1P^o_1$	0	1206.51	1.67
P IV	$2p^63s^2 - 2p^63s3p$	$^1S_0$	$^1P^o_1$	0	950.655	1.60
Si II	$3s^23p - 3s^24s$	$^2P^o_{1/2}$	$^2S_{1/2}$	0	1526.72	0.133
		$^2P^o_{3/2}$	$^2S_{1/2}$	287.24	1533.45	0.133
P III	$3s^23p - 3s3p^2$	$^2P^o_{1/2}$	$^2D_{3/2}$	0	1334.808	0.029
		$^2P^o_{3/2}$	$^2D_{5/2}$	559.14	1344.327	0.026
Si I	$3s^23p^2 - 3s^23p4s$	$^3P_0$	$^3P^o_1$	0	2515.08	0.17
		$^3P_1$	$^3P^o_2$	77.115	2507.652	0.0732
		$^3P_2$	$^3P^o_2$	223.157	2516.870	0.115
P II	$3s^23p^2 - 3s3p^3$	$^3P_0$	$^3P^o_1$	0	1301.87	0.038
		$^3P_1$	$^3P^o_2$	164.9	1305.48	0.016
		$^3P_2$	$^3P^o_2$	469.12	1310.70	0.115
S III	$3s^23p^2 - 3s3p^3$	$^3P_0$	$^3D^o_1$	0	1190.206	0.61
		$^3P_1$	$^3D^o_2$	298.69	1194.061	0.46
		$^3P_2$	$^3D^o_3$	833.08	1200.07	0.51
Cl IV	$3s^23p^2 - 3s3p^3$	$^3P_0$	$^3D^o_1$	0	973.21	0.55
		$^3P_1$	$^3D^o_2$	492.0	977.56	0.41
		$^3P_2$	$^3D^o_3$	1341.9	984.95	0.47
P I	$3s^23p^3 - 3s^23p^24s$	$^4S^o_{3/2}$	$^4P_{5/2}$	0	1774.951	0.154
S II	$3s^23p^3 - 3s^23p^24s$	$^4S^o_{3/2}$	$^4P_{5/2}$	0	1259.518	0.12
Cl III	$3s^23p^3 - 3s^23p^24s$	$^4S^o_{3/2}$	$^4P_{5/2}$	0	1015.019	0.58
S I	$3s^23p^4 - 3s^23p^34s$	$^3P_2$	$^3S^o_1$	0	1807.311	0.11
		$^3P_1$	$^3S^o_1$	396.055	1820.343	0.11
		$^3P_0$	$^3S^o_1$	573.640	1826.245	0.11
Cl II	$3s^23p^4 - 3s3p^5$	$^3P_2$	$^3P^o_2$	0	1071.036	0.014
		$^3P_1$	$^3P^o_2$	696.00	1079.080	0.00793
		$^3P_0$	$^3P^o_1$	996.47	1075.230	0.019
Cl I	$3s^23p^5 - 3s^23p^44s$	$^2P^o_{3/2}$	$^2P_{3/2}$	0	1347.240	0.114
		$^2P^o_{1/2}$	$^2P_{3/2}$	882.352	1351.657	0.0885
Ar II	$3s^23p^5 - 3s3p^6$	$^2P^o_{3/2}$	$^2S_{1/2}$	0	919.781	0.0089
		$^2P^o_{1/2}$	$^2S_{1/2}$	1431.583	932.054	0.0087
Ar I	$3p^6 - 3p^54s$	$^1S_0$	$^2[1/2]^o$	0	1048.220	0.25

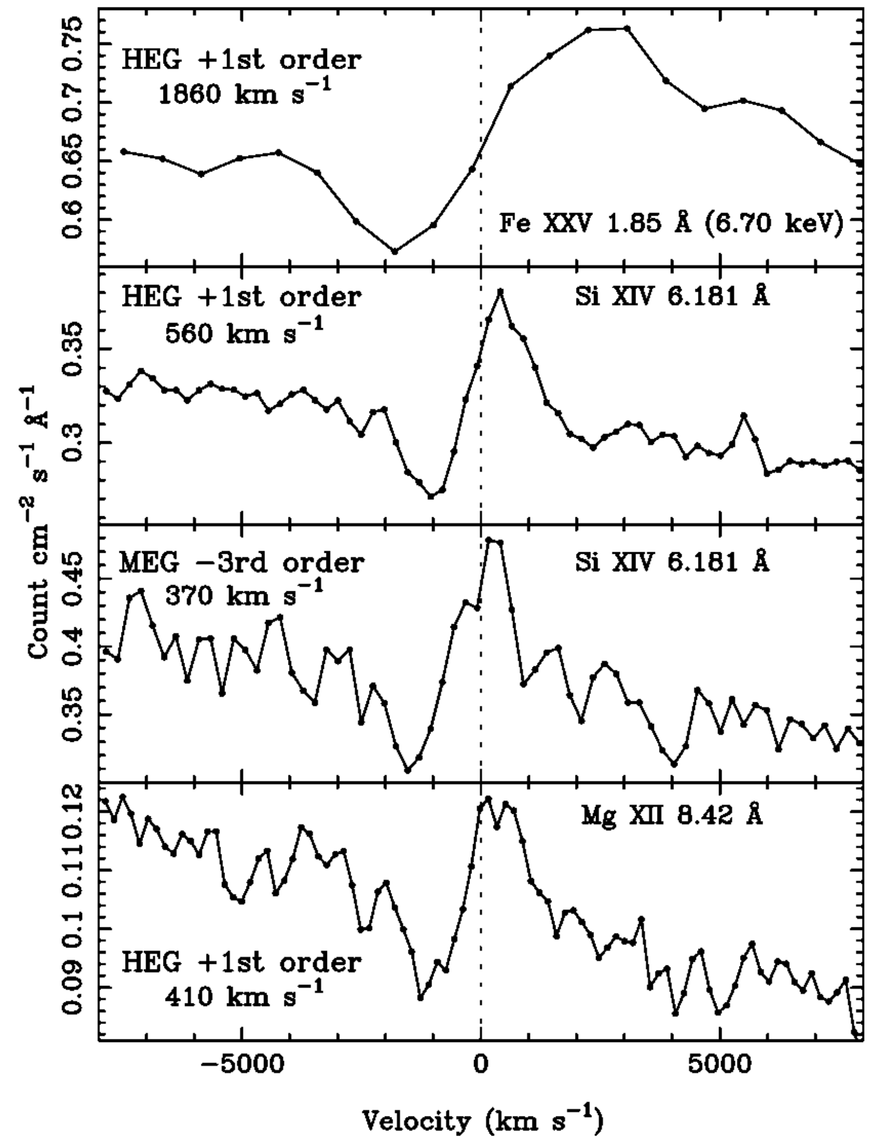
[a] Transition data from NIST Atomic Spectra Database v4.0.0 (Ralchenko et al. 2010)

P Cygni Profile

- The PCygni profile is characterized by strong emission lines with corresponding blueshifted absorption line.



zeta Puppis (Snow et al., 1994, ApJS, 95, 163)

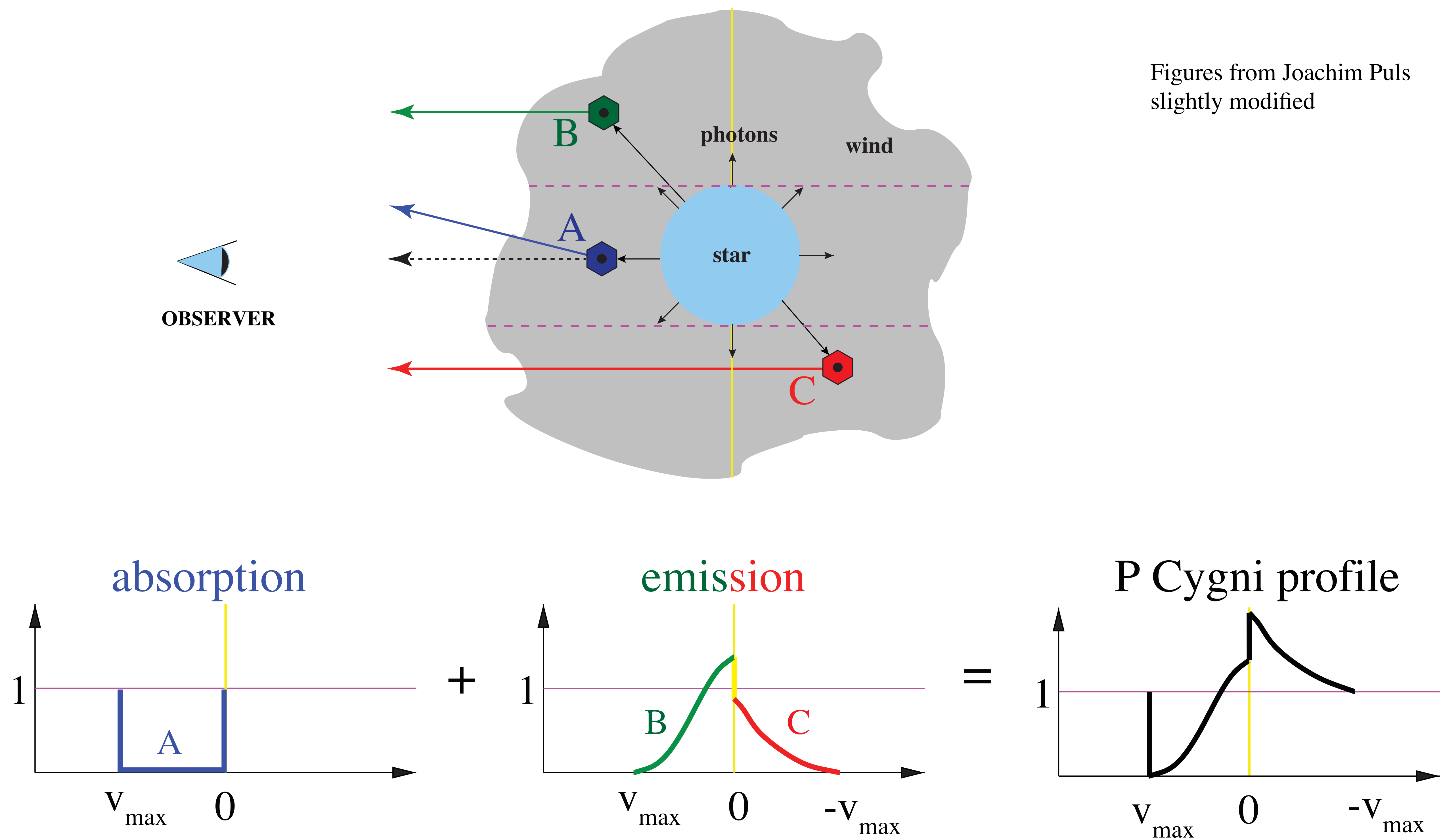


Circinus X-1
(Brandt & Schulz, 2000, ApJ, 544, L123)

P Cygni profile formation

- The blueshifted absorption line is produced by material moving away from the star and toward us, whereas the emission come from other parts of the expanding shell.

Figures from Joachim Puls slightly modified



Lyα Resonance Scattering

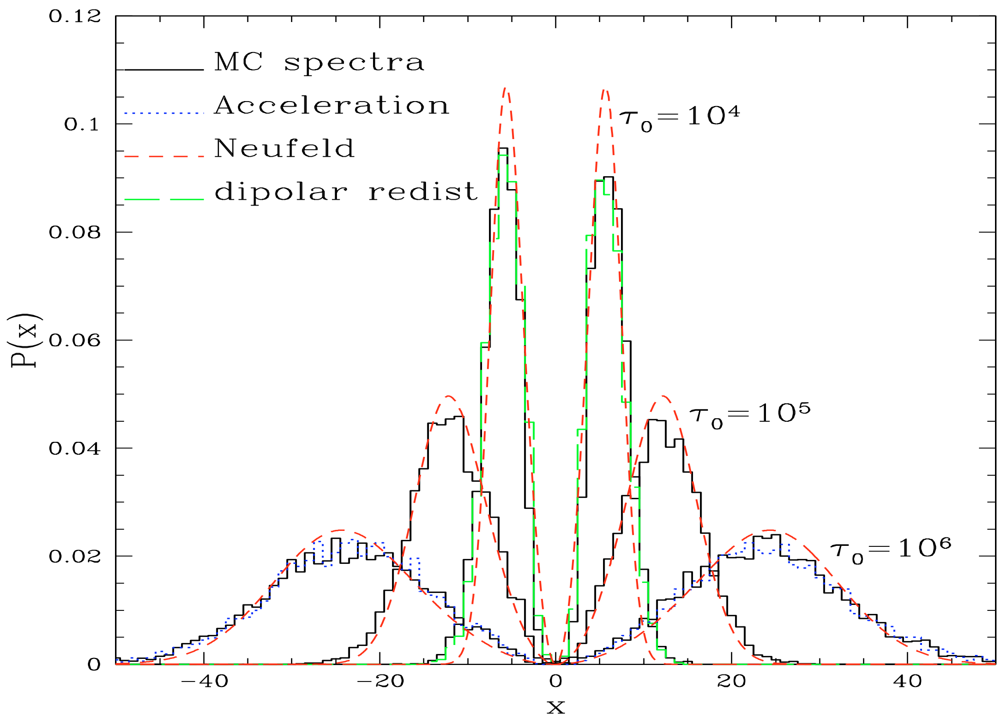
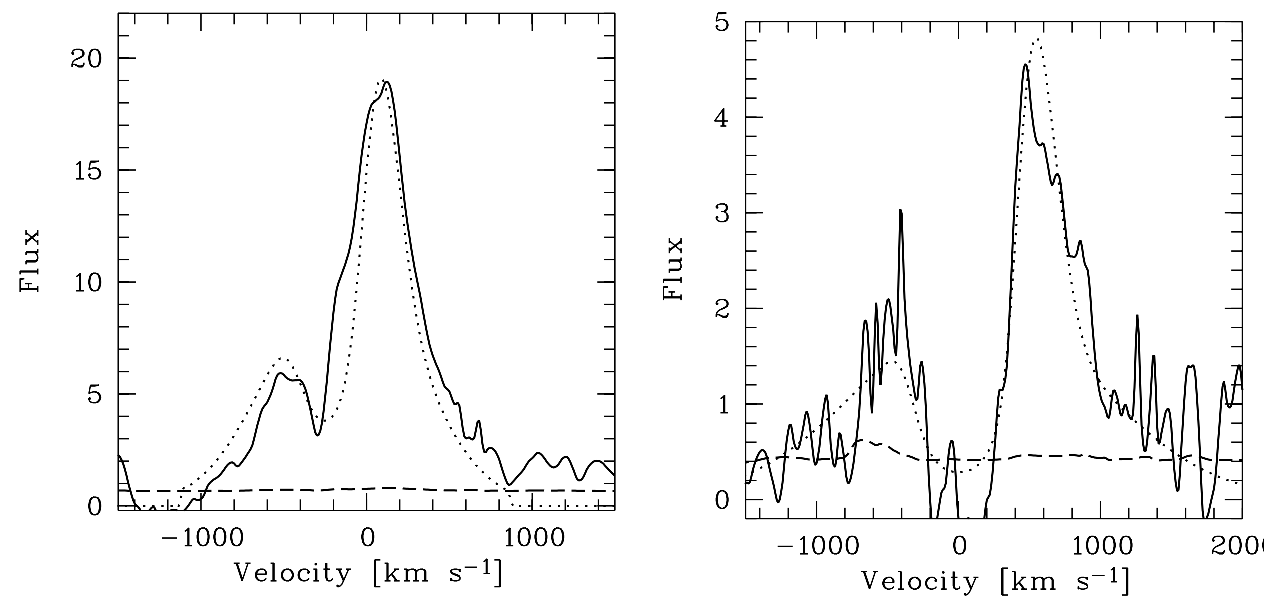


Fig. 1. Predicted emergent Lyα profiles for monochromatic line radiation emitted in a dust-free slab of different optical depths (solid lines) compared with analytic solutions from Neufeld (1990, dashed). The dotted blue curve shows the line profile obtained using a frequency redistribution function, which skips a large number of resonant core scatterings. The adopted conditions of the medium are: T = 10 K (i.e. $a = 1.5 \times 10^{-2}$) and $\tau_0 = 10^4, 10^5, 10^6$ *from top to bottom*. The green long-dashed curve, obtained with a dipolar angular redistribution, overlaps perfectly the black solid line obtained with the isotropic angular redistribution function, illustrating the fact that in static media, isotropy is a very good approximation.

Verhamme et al. (2006, A&A, 460, 397)



(a) FDF-5215 (b) FDF-7539

ID	v_{dis}(core) [km s^{-1}]	v_{dis}(shell) [km s^{-1}]	N_{HI} [cm^{-2}]	$v_{outflow}$ [km s^{-1}]	z
4691	600	60	4×10^{17}	12	3.30
5215	500	125	$<2 \times 10^{16}$	125	3.15
7539	1140	190	2.5×10^{16}	190	3.29

Comparizon of the observed Lyα lines (solid lines) and the best-fit theoretical models. The dashed line indicates the noise level of the observed spectrum.

Tapken et al. (2007, A&A, 467, 63)

Homework

- Solve the problem 3.2 for the cyclotron or gyro radiation (nonrelativistic version of the synchrotron radiation)