Radiative Processes in Astrophysics

Lecture 11

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[Sunyaev-Zeldovich effect]

- The **Sunyaev-Zeldovich effect** is the distortion of the blackbody spectrum (T = 2.73 K) of the CMB owing to the IC scattering of the CMB photons by the energetic electrons in the galaxy clusters.

  <u>Thermal SZ effects</u>, where the CMB photons interact with electrons that have high energies due to their temperature.

  <u>Kinematic SZ effects (Ostriker-Vishniac effect)</u>, a second-order effect where the CMB photons interact with electrons that have high energies due to their bulk motion (peculiar motion). The motions of galaxies and clusters of galaxies relative to the Hubble flow are called peculiar velocities. The plasma electrons in the cluster also have this velocity. The energies of the CMB photons that scattered by the electrons reflect this motion.

  Determinations of the peculiar velocities of clusters enable astronomers to map out the growth of large-scale structure in the universe. This topic is fundamental importance, and the kinetic SZ effect is a promising method for approaching it.

- Thermal SZ effect

- The net effect of the IC scattering on the photon spectrum is obtained by multiplying the photon number spectrum by the kernel $K(\nu/\nu_0)$ and integrating over the spectrum.

$$N_{scatt}(\nu) = \int_0^\infty N(\nu_0) K(\nu/\nu_0) d\nu_0$$

The net effect is that the BB spectrum is shifted to the right and distorted.

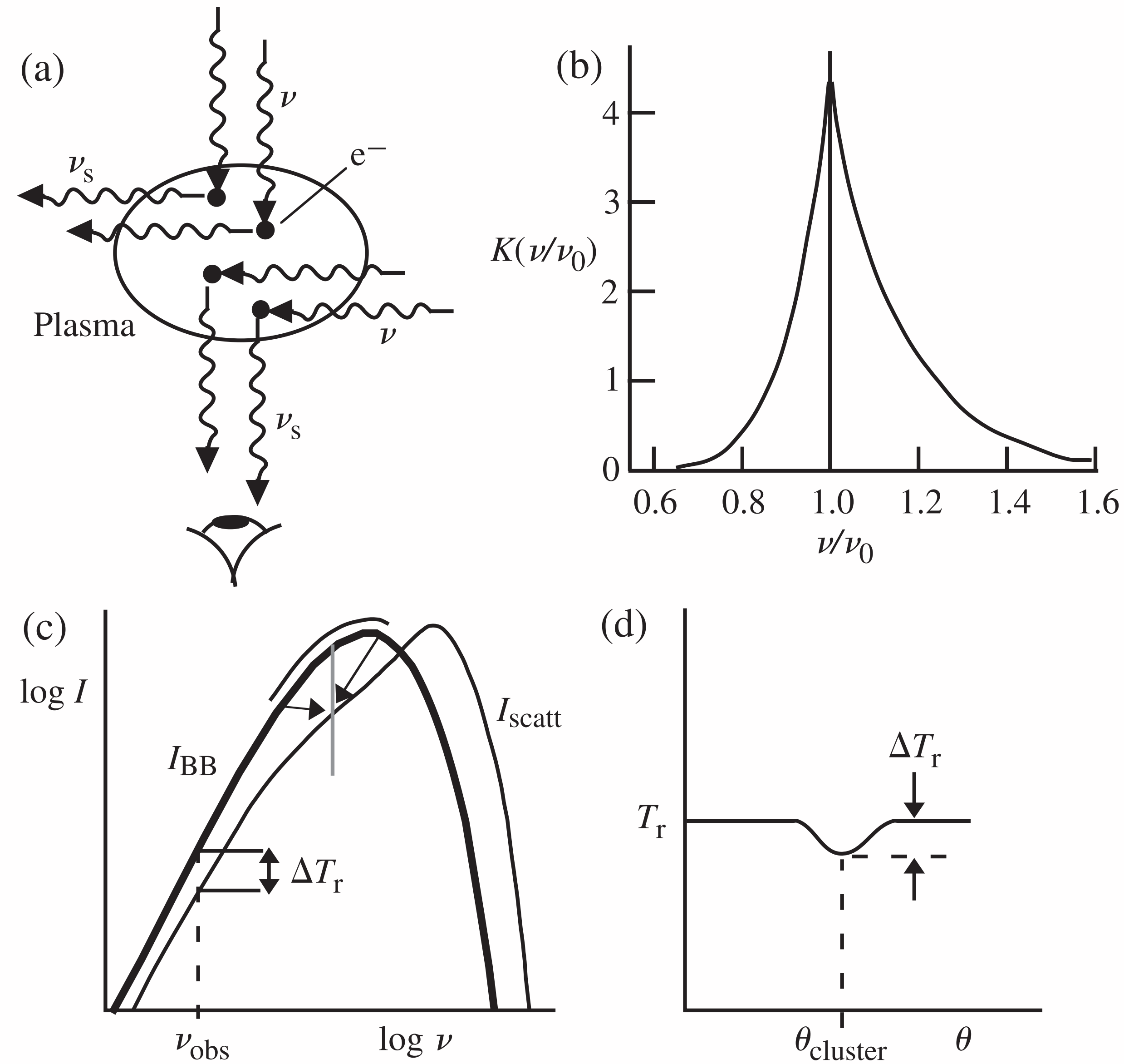
Observations of the CMB are most easily carried out in the low-frequency Rayleigh-Jeans region of the spectrum $(h\nu \ll kT_{CMB})$.

Measurement of the CMB temperature as a function of position on the sky would thus exhibit antenna temperature dips in the directions of clusters that contain hot plasmas.

Note that the scattered spectrum is not a BB spectrum. The effect temperature increases. But, the total number of photons detected in a given time over the entire spectrum remains constant.

The result of such scatterings for an initial blackbody photon spectrum is shown in the following figure for the value:

$$(kT_e/mc^2)\tau = 0.15$$



- Change of the BB temperature

In the Rayleigh-Jeans region,

$$I(\nu) = \frac{2\nu^2}{c^2} k_{\mathrm{B}} T_{\mathrm{CMB}}$$

If the spectrum is shifted parallel to itself on a log-log plot, the fractional frequency change of a scattered photon is constant.

$$\varepsilon = \frac{\Delta\nu}{\nu} = \frac{\nu' - \nu}{\nu} = \text{constant} \quad \text{or} \quad \nu' = \nu(1+\varepsilon) \longrightarrow d\nu' = d\nu(1+\varepsilon)$$

Total photon number is conserved: $N'(\nu')d\nu' = N(\nu)d\nu \rightarrow \frac{I'(\nu')}{h\nu'} d\nu' = \frac{I(\nu)}{h\nu} d\nu$

$$I'(\nu') = I(\nu) = I\left(\frac{\nu'}{1+\varepsilon}\right) = \frac{2\nu'^2}{c^2(1+\varepsilon)^2} k_{\mathrm{B}} T_{\mathrm{CMB}}$$

$$\frac{\Delta I}{I} = \frac{I' - I}{I} = \frac{1}{(1+\varepsilon)^2} - 1 \approx -2\varepsilon = -2\frac{\Delta\nu}{\nu}$$

$$\frac{\Delta T_{\mathrm{CMB}}}{T_{\mathrm{CMB}}} = \frac{\Delta I}{I} \approx -2\varepsilon = -2\frac{\Delta\nu}{\nu}$$

The properly calculated result is $\varepsilon = \frac{\Delta\nu}{\nu} = \frac{k_{\mathrm{B}} T_{\mathrm{CMB}}}{mc^2}\tau$. $\longrightarrow \frac{\Delta T_{\mathrm{CMB}}}{T_{\mathrm{CMB}}} \approx -2\frac{k_{\mathrm{B}} T_{\mathrm{CMB}}}{mc^2}\tau$

A typical cluster have
an average electron density of $\sim 2.5 \times 10^{-3}$ cm^{-3} ,
a core radius of $R_c \sim 10^{24}$ cm (~ 320 kpc) ,
and an electron temperature of $k_{\rm B}T_e \approx 5$ keV.

A typical optical depth is thus

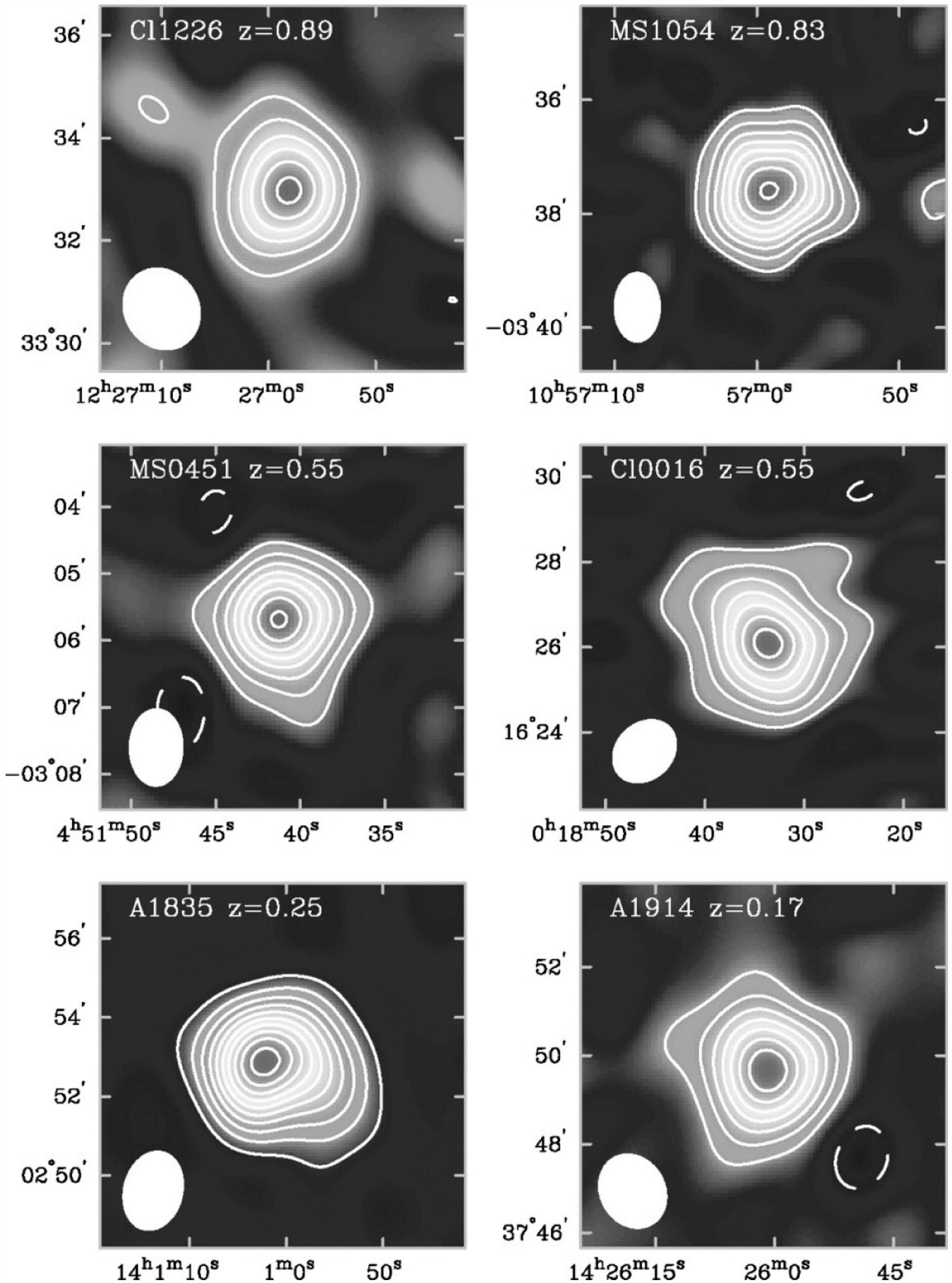
$$\tau \approx 3\sigma_T n_e R_c \approx 0.005$$

The expected antenna temperature change is

$$\frac{\Delta T_{\rm CMB}}{T_{\rm CMB}} \approx -1\times 10^{-4}$$

$$\Delta T_{\rm CMB} \approx -0.3 \text{ mK} \quad \text{for} \quad T_{\rm CMB} = 2.7 \text{ K}$$

This effect has been measured in dozens of clusters.



Interferometric images at 30 GHz of size clusters of galaxies. The solid white contours indicate negative decrements to the CMB. (Carlstrom et al. 2002, ARAA, 40, 643)

- Hubble Constant

- A value of the Hubble constant is obtained for a given galaxy only if one has independent measures of a recession speed v and a distance d of a galaxy.

$$H_0 = \frac{v}{d}$$

Recession speed is readily obtained from the spectral redshift

Distance:

X-ray observations:

$$I(\nu, T_e) = C\frac{g(\nu, T_e)}{T_e^{1/2}}\exp(-h\nu / kT_e)n_e^2 2R$$

CMB decrement:

$$\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} = -2\frac{kT_e}{mc^2}\tau = -2\frac{kT_e}{mc^2}(\sigma_T n_e 2R)$$

The radio and X-ray measurements yield absolute values of the electron density n_e and cluster radius R without a priori knowledge of the cluster distance.

Imaging of the cluster in the radio or X-ray band yields the angular size of the cluster θ. Then the distance d to the cluster is obtained by

$$d = \frac{R}{\theta}$$

The SZ effect (at radio frequencies) in conjunction with X-ray measurements can give distances to clusters of galaxies.

[Kompaneets Equation]

- The Kompaneets equation describes the time evolution of the distribution of photon occupancies in the case where photons and electrons are interacting through Compton scattering.
- Boltzmann transport equation

$$\frac{\partial n(\omega)}{\partial t} = c\int d^3p \int d\Omega \frac{d\sigma}{d\Omega}\left[f_e(\mathbf{p}')n(\omega')(1+n(\omega)) - f_e(\mathbf{p})n(\omega)(1+n(\omega'))\right]$$

In $1+n(\omega)$, the "1" for spontaneous Compton scattering, and the $n(\omega)$ for stimulated Compton scattering.

The Boltzmann equation may be expanded to second order in the small energy transfer, yielding an approximation called the *Fokker-Plank equation*. For photons scattering off a nonrelativistic, thermal distribution of electrons, the Fokker-Plank equation was first derived by A. S. Kompaneets (1957) and is known as the Kompaneets equation.

$$\frac{\partial n}{\partial t_c} = \left(\frac{k_B T}{mc^2}\right)\frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n + n^2\right)\right] \qquad \text{where} \quad x \equiv \frac{\hbar\omega}{k_B T}, \quad \text{and} \quad t_c \equiv (n_e \sigma_T c)t$$

- For the complete derivation, see the books "X-ray spectroscopy in Astrophysics (eds. van Paradijs)", pages 213-218, and the book "High Energy Astrophysics (Katz)", pages 103-110.
- Monte Carlo Simulation of Compton scattering: see "Pozdnyakov, Sobol, and Suyaev (1983, Soviet Scientific Reviews, vol. 2, 189-331)" (1983ASPRv...2..189P)

8. Plasma Effects

[Dispersion in Cold, Isotropic Plasma]

- Roughly speaking a plasma is a

  globally neutral

  partially or completely ionized gas

- **<u>Plasma Dispersion Relation</u>**

  Assume a space and time variation of all quantities of the form $\exp\left[i\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right]$.

$$\begin{aligned}
\nabla\cdot\mathbf{E} &= 4\pi\rho \\
\nabla\cdot\mathbf{B} &= 0 \\
\nabla\times\mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \\
\nabla\times\mathbf{B} &= \frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}
\end{aligned}
\longrightarrow
\begin{aligned}
i\mathbf{k}\cdot\mathbf{E} &= 4\pi\rho \\
i\mathbf{k}\cdot\mathbf{B} &= 0 \\
i\mathbf{k}\times\mathbf{E} &= i\frac{\omega}{c}\mathbf{B} \\
i\mathbf{k}\times\mathbf{B} &= \frac{4\pi}{c}\mathbf{j}-i\frac{\omega}{c}\mathbf{E}
\end{aligned}$$

Equation of motion when there is no external magnetic field

$$m\dot{\mathbf{v}} = -e\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)\approx -e\mathbf{E} \longrightarrow \begin{aligned} -im\omega\mathbf{v} &= -e\mathbf{E} \\ \mathbf{v} &= \frac{e\mathbf{E}}{i\omega m}\end{aligned}$$

<u>Current density:</u>

$$\mathbf{j}\equiv -ne\mathbf{v}=\sigma\mathbf{E}$$

conductivity: $\sigma\equiv\dfrac{ine^2}{\omega m}$

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 \qquad -i\omega\rho + i\mathbf{k}\cdot\mathbf{j} = 0 \;\rightarrow\; \rho = \frac{1}{\omega}\mathbf{k}\cdot\mathbf{j} = \frac{\sigma}{\omega}\mathbf{k}\cdot\mathbf{E}$$

Using

$$\mathbf{j} = \sigma\mathbf{E}$$

$$\rho = \frac{\sigma}{\omega}\mathbf{k}\cdot\mathbf{E}$$

$$i\mathbf{k}\cdot\mathbf{E} = 4\pi\rho$$
$$i\mathbf{k}\cdot\mathbf{B} = 0$$
$$i\mathbf{k}\times\mathbf{E} = i\frac{\omega}{c}\mathbf{B}$$
$$i\mathbf{k}\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j} - i\frac{\omega}{c}\mathbf{E}$$

$\longrightarrow$

$$i\mathbf{k}\cdot\epsilon\mathbf{E} = 0$$
$$i\mathbf{k}\cdot\mathbf{B} = 0$$
$$i\mathbf{k}\times\mathbf{E} = i\frac{\omega}{c}\mathbf{B}$$
$$i\mathbf{k}\times\mathbf{B} = -i\frac{\omega}{c}\epsilon\mathbf{E}$$

where

dielectric constant:

$$\epsilon \equiv 1 - \frac{4\pi\sigma}{i\omega} = 1 - \frac{4\pi n e^2}{m\omega^2}$$
$$= 1 - \left(\frac{\omega_p}{\omega}\right)^2$$

plasma frequency:

$$\omega_p^2 \equiv \frac{4\pi n e^2}{m}$$

$$\omega_p = 5.63\times 10^4 \left(n/\mathrm{cm}^3\right)^{1/2} \quad \mathrm{s}^{-1}$$

These equations are now "source-free."

k, **E**, **B** form a mutually orthogonal right-hand vector triad.

the dispersion relation is

$$c^2k^2 = \epsilon\omega^2$$

or $c^2k^2 = \omega^2 - \omega_p^2$

$$\boxed{\omega^2 = \omega_p^2 + c^2k^2}$$

- If $\omega < \omega_p$, then $k^2 < 0$. Wavenumber is imaginary!

$$k = \frac{i}{c}\sqrt{\omega_p^2 - \omega^2}$$

The wave amplitude decreases as $e^{-|k|r}$ on a scale of the order of $\approx 2\pi c / \omega_p$.

Thus ω_p defines a plasma cutoff frequency below which there is no electromagnetic propagation.

For instance, Earth ionosphere prevents extraterrestrial radiation at frequencies less than about 1 MHz from being observed at the earth's surface ($n \sim 10^4$ cm^{-3}).

Method of probing the ionosphere:

Let a pulse of radiation in a narrow range about ω be directed straight upward from the earth's surface.

When there is a layer at which n is large enough to make $\omega_p > \omega$, the pulse will be totally reflected from the layer.

The time delay of the pulse provides information on the height of the layer.

By making such measurements at many different frequencies, the electron density can be determined as a function of height.

- **Group and Phase Velocity**

When $\omega > \omega_p$, waves do propagate without damping.

Phase velocity $\quad v_{\rm ph} \equiv \dfrac{\omega}{k} = \dfrac{c}{n_r}$

where the index of refraction $\quad n_r \equiv \sqrt{\epsilon} = \sqrt{1 - \dfrac{\omega_p^2}{\omega^2}} \;\rightarrow\; v_{\rm ph} > c$

Group velocity

$$\omega^2 = \omega_p^2 + c^2 k^2 \quad \longrightarrow \quad v_{\rm g} \equiv \frac{\partial \omega}{\partial k} = \frac{k}{\omega} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \;\rightarrow\; v_{\rm g} < c$$

- **Pulsar**

Each small range of frequencies travels at a slightly different group velocity and will reach earth at a slightly different time.

Let d = pulsar distance.

the time required for a pulse to reach earth at frequency ω is $\quad t_p = \int_0^d \dfrac{ds}{v_{\rm g}}$.

The plasma frequency in ISM is usually quite low ($\sim 10^3$ Hz), so we can assume that $\omega \gg \omega_p$.

$$\frac{1}{v_{\rm g}} = \frac{1}{c}\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1/2} \approx \frac{1}{c}\left(1 + \frac{1}{2}\frac{\omega_p^2}{\omega^2}\right) \;\rightarrow\; t_p \approx \frac{d}{c} + \frac{2\pi e^2}{cm\omega^2}\int_0^d n\, ds$$

transit time for a vacuum + plasma correction

- **<u>Dispersion measure</u>**:

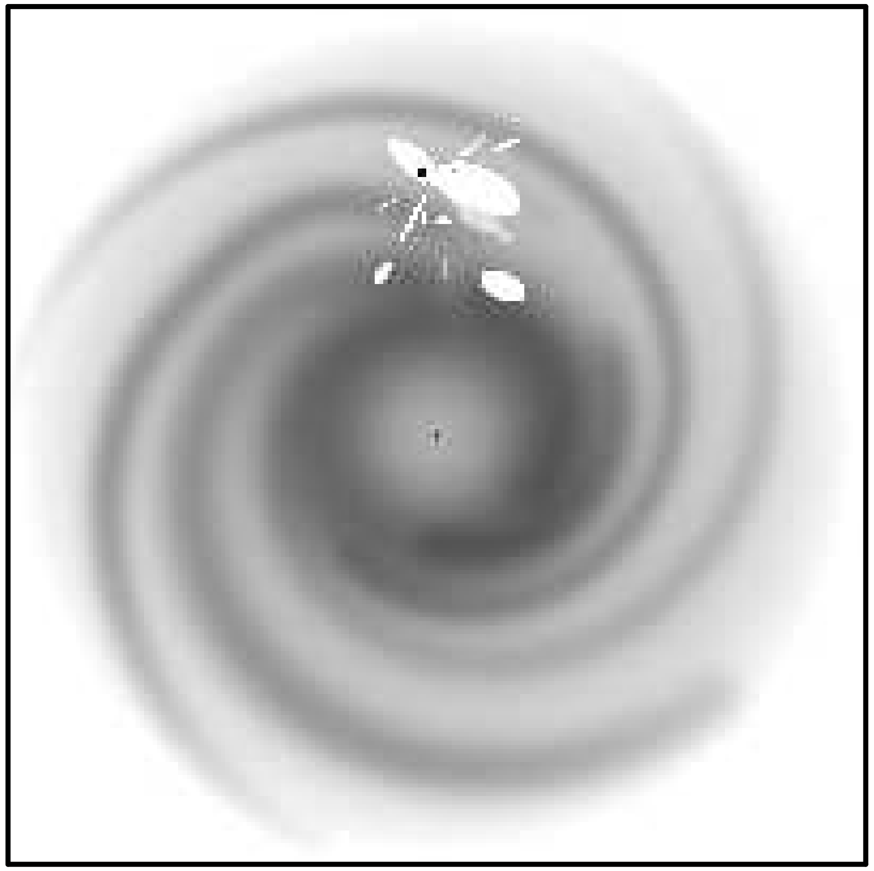
the rate of change of arrival time w.r.t.: $\frac{dt_p}{d\omega} = -\frac{4\pi e^2}{cm\omega^3}\mathcal{DM}$ where $\mathcal{DM} \equiv \int_0^d n\,ds$

If one has idea of pulsar distance, one can use pulsar data to map free electron density.

Taylor & Cordes (1993, ApJ, 411, 674)

Cordes & Lazio (2003, arXiv:astro-ph/0207156)

Schnitzeler (2012, MNRAS, 427, 664)



Cordes & Lazio (2003)

FIG. 2.— Electron density corresponding to the best fit model plotted as a grayscale with logarithmic levels on a 30×30 kpc x-y plane at z=0 and centered on the Galactic center. The most prominent large-scale features are the spiral arms, a thick, tenuous disk, a molecular ring component. A Galactic center component appears as a small dot. The small-scale, lighter features represent the local ISM and underdense regions required for by some lines of sight with independent distance measurements. The small dark region embedded in one of the underdense, ellipsoidal regions is the Gum Nebula and Vela supernova remnant.

[Propagation along a Magnetic Field; Faraday Rotation]

- Now we consider the effect of an external, fixed magnetic field $\mathbf{B}_0$.

  The properties of the waves will then depend on the direction of propagation relative to the magnetic field direction.

$$\omega_B = \frac{eB_0}{mc} = 1.67\times 10^7 \left(B_0\,/\,\mathrm{G}\right)\ \mathrm{s}^{-1}$$
$$\hbar\omega_B = 1.16\times 10^{-8}\left(B_0\,/\,\mathrm{G}\right)\ \mathrm{eV}$$

  If the fixed magnetic field $\mathbf{B}_0$ is much stronger than the field strengths of the propagating wave, then the equation of motion of an electron is approximately

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - \frac{e}{c}\mathbf{v}\times\mathbf{B}_0$$

  For simplicity, assume that the wave propagates along the fixed field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, and assume that the wave is circularly polarized and sinusoidal.

$$\mathbf{E}(t) = Ee^{-i\omega t}\left(\hat{\mathbf{x}} \mp i\hat{\mathbf{y}}\right) \ \rightarrow\ \frac{d\mathbf{v}_\parallel}{dt} = 0,\ \ \mathbf{v}_\parallel = \text{constant}$$

Here, $\mp$ denotes right and left circular polarization.

$$\mathbf{v}_\perp = v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}}$$

$$(-i\omega)e^{-i\omega t}\left(v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}}\right) = -\frac{e}{m}Ee^{-i\omega t}\left(\hat{\mathbf{x}} \mp i\hat{\mathbf{y}}\right) - \frac{e}{mc}e^{-i\omega t}\left(v_yB_0\hat{\mathbf{x}} - v_xB_0\hat{\mathbf{y}}\right)$$

$$(-i\omega)e^{-i\omega t}\left(v_x\hat{\mathbf{x}}+v_y\hat{\mathbf{y}}\right)=-\frac{e}{m}Ee^{-i\omega t}\left(\hat{\mathbf{x}}\mp i\hat{\mathbf{y}}\right)-\frac{e}{mc}e^{-i\omega t}\left(v_yB_0\hat{\mathbf{x}}-v_xB_0\hat{\mathbf{y}}\right)$$

$$v_x=-\frac{ie}{m\omega}E-\frac{ieB_0}{mc\omega}v_y,\;\; v_y=\mp\frac{e}{m\omega}E+\frac{ieB_0}{mc\omega}v_x$$

$$v_x=-\frac{ie}{m\omega}E-\frac{ieB_0}{mc\omega}\left(\mp\frac{e}{m\omega}E+\frac{ieB_0}{mc\omega}v_x\right)\;\rightarrow\;\left(1-\frac{e^2B_0{}^2}{m^2c^2\omega^2}\right)v_x=-\frac{ie}{m\omega}E\left(1\mp\frac{eB_0}{mc\omega}\right)$$

$$\rightarrow\;\left(\omega^2-\omega_B^2\right)v_x=-\frac{ie}{m}E\left(\omega\mp\omega_B\right)$$

$$\rightarrow\;v_x=-\frac{ie}{m}E\,\frac{1}{\omega\pm\omega_B}$$

$$v_y=\mp\frac{e}{m\omega}E+\frac{ieB_0}{mc\omega}\left(-\frac{ie}{m\omega}E-\frac{ieB_0}{mc\omega}v_y\right)\;\rightarrow\;\left(1-\frac{e^2B_0{}^2}{m^2c^2\omega^2}\right)v_y=\mp\frac{e}{m\omega}E\left(1\mp\frac{eB_0}{mc\omega}\right)$$

$$\rightarrow\left(\omega^2-\omega_B^2\right)v_y=\mp\frac{e}{m}E\left(\omega\mp\omega_B\right)$$

$$\rightarrow v_y=\mp\frac{e}{m}E\frac{1}{\omega\pm\omega_B}$$

current density, conductivity

$$\therefore\;\mathbf{v}=\frac{-ie}{m\left(\omega\pm\omega_B\right)}\mathbf{E}(t)\quad\rightarrow\quad\mathbf{j}\equiv-ne\mathbf{v}=\sigma\mathbf{E},\;\;\sigma_{\mathrm{R,L}}\equiv\frac{ine^2}{m\left(\omega\pm\omega_B\right)}$$

Dielectric constant

$$\epsilon \equiv 1 - \frac{4\pi\sigma}{i\omega} = 1 - \frac{4\pi n e^2}{m\omega(\omega \pm \omega_B)}$$

$$\epsilon_{\mathrm{R,L}} = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_B)}$$

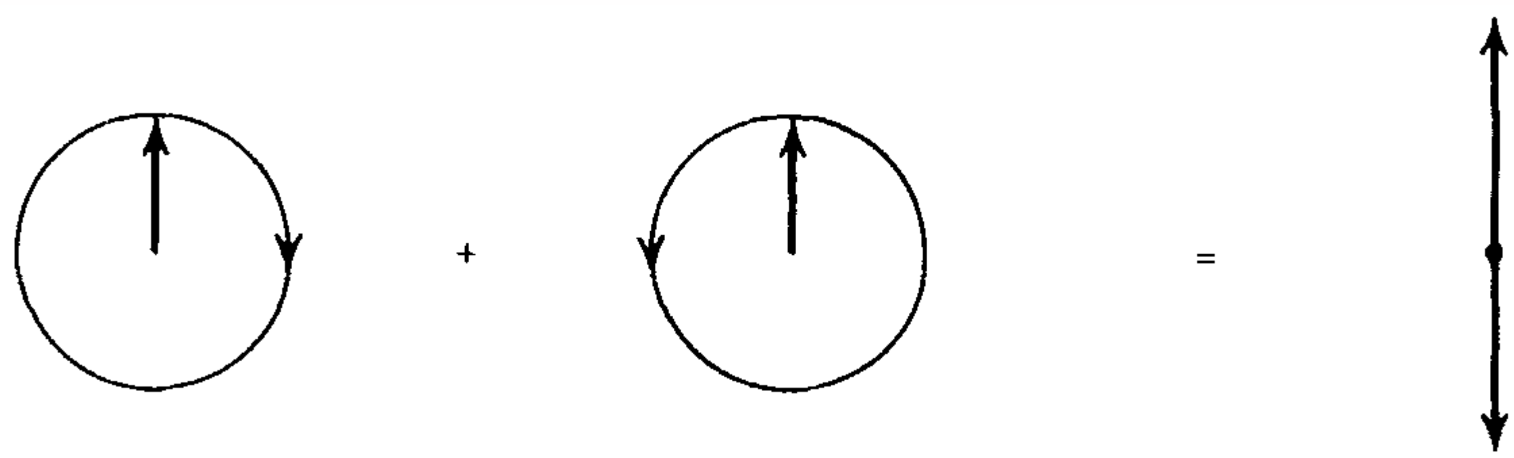
Right (+) and left(-) circularly polarized waves travel with different speeds.

Speed difference sense is $v_{\mathrm{R}} > v_{\mathrm{L}}$.

- **Faraday Rotation**

If the incident radiation is ***circularly polarized*** (either R or L), then the radiation will encounter different dispersion than unmagnetized case. But, the radiation will still remain circularly polarized.

If the incident radiation is ***linearly polarized***, i.e., a linear superposition of a right-hand and a left-hand polarized wave, then ***the line of polarization will rotate as it propagates***. This effect is called Faraday rotation.



- Phase after traveling a distance *d*:

$$\phi_{R,L} = \int_0^d k_{R,L}\, dz$$

Assume that $\omega \gg \omega_p, \ \omega \gg \omega_B$

$$k_{R,L} = \frac{\omega}{c}\sqrt{\epsilon_{R,L}} = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2\left(1 \pm \omega_B/\omega\right)}\right] \approx \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{2\omega^2}\left(1 \mp \frac{\omega_B}{\omega}\right)\right]$$

$$= \frac{\omega}{c} - \frac{\omega_p^2}{2c\omega} \pm \frac{\omega_p^2\omega_B}{2c\omega^2} = k_0 \pm \Delta k$$

Consider an electromagnetic wave that starts off linearly polarized in the *x*-direction at the source.

$$\mathbf{E}(t) = Ee^{-i\omega t}\hat{\mathbf{x}} = \frac{1}{2}\left[\left(\hat{\mathbf{x}} - i\hat{\mathbf{y}}\right) + \left(\hat{\mathbf{x}} + i\hat{\mathbf{y}}\right)\right]Ee^{-i\omega t}$$

After propagating a distance d through a magnetized plasma toward the observer, the electric field will be

$$\mathbf{E}(t) = \frac{1}{2}\left[\left(\hat{\mathbf{x}} - i\hat{\mathbf{y}}\right)e^{i(\phi+\varphi)} + \left(\hat{\mathbf{x}} + i\hat{\mathbf{y}}\right)e^{i(\phi-\varphi)}\right]Ee^{-i\omega t} \qquad \text{where } \int_0^d k_{R,L}\, dz = \int_0^d k_0\, dz \pm \int_0^d \Delta k\, dz \equiv \phi \pm \varphi$$

$$= \left(\hat{\mathbf{x}}\cos\varphi + \hat{\mathbf{y}}\sin\varphi\right)Ee^{i(\phi-\omega t)}$$

Radiation that starts linearly polarized in a certain direction is rotated by the *Faraday effect* through an angle φ after propagating a distance *d* through a magnetized plasma.

$$\varphi = \int_0^d \Delta k\, dz = \frac{1}{2}\int_0^d \frac{\omega_p^2\omega_B}{c\omega^2}\, ds = \frac{2\pi e^3}{m^2c^2\omega^2}\int_0^d nB_{\parallel}\, ds$$

- We cannot, of course, generally measure the absolute rotation angle, since we do not know the intrinsic polarization direction of the radiation when it started from the source.

  However, since φ varies with frequency (as ω^{-2}), we can determine the value of integral $\int nB_{\parallel}\,ds$ by making measurements at several frequencies. This can give information about the interstellar magnetic field.

  Rotation measure is defined by $\varphi = \dfrac{2\pi e^3}{m^2c^2\omega^2}\mathcal{RM} = \dfrac{e^3\lambda^2}{2\pi m^2c^4}\mathcal{RM}$, where $\mathcal{RM} \equiv \int_0^d nB_{\parallel}\,ds$

  However, the field changes direction often along the line of sight and this method gives only a lower limit to actual field magnitudes.

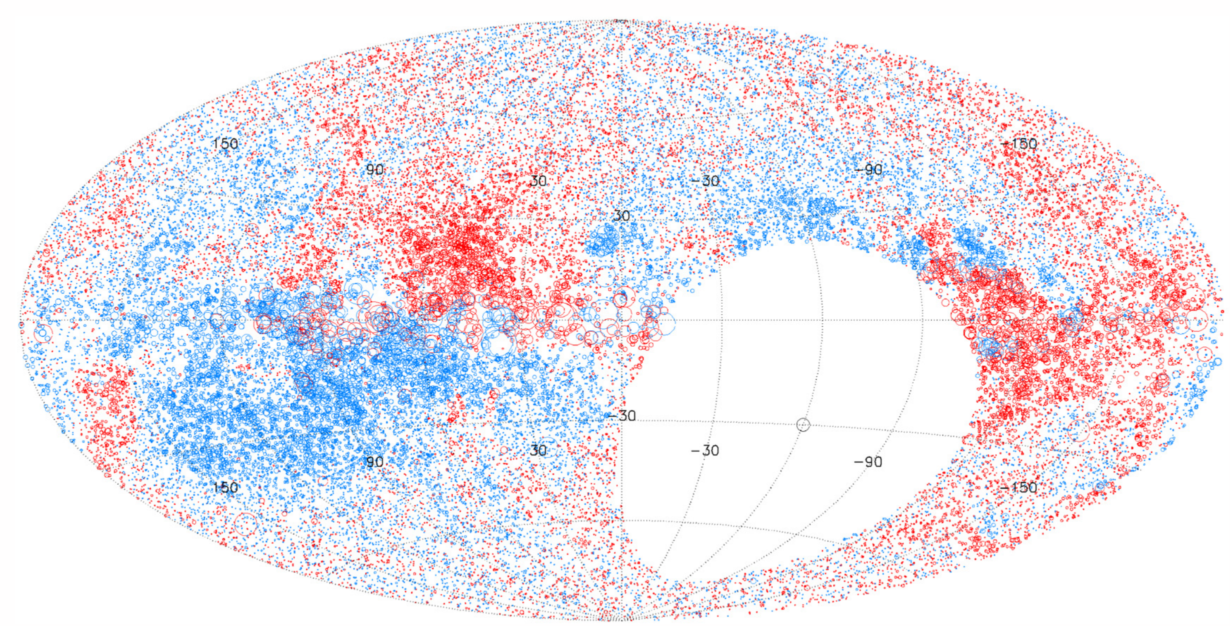
For measurements toward sources (pulsars) where the dispersion measure (DM) is also known, we can derive an estimate of the mean field strength along the line of sight.

$$\langle B_{\parallel}\rangle = \frac{\mathcal{RM}}{\mathcal{DM}}$$

Radio astronomers have concluded that

$$\langle n_e\rangle \approx 0.03\ \mathrm{cm}^{-3}$$

$$\langle B_{\parallel}\rangle \approx 3\ \mu\mathrm{G}$$



(Taylor, Stil, & Sunstrum 2009, ApJ, 702, 1230)
Red circles are positive RM and blue circles are negative.
The size of the circle scales linearly with magnitude of RM.

[Plasma Effects in High-Energy Emission Processes]

- Maxwell equations in dielectric medium:

$$\nabla \cdot (\epsilon \mathbf{E}) = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial (\epsilon \mathbf{E})}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

These equations formally result from Maxwell's equation in vacuum by the substitutions.

$$\mathbf{E} \rightarrow \sqrt{\epsilon}\mathbf{E}$$

$$c \rightarrow c/\sqrt{\epsilon}$$

$$\mathbf{B} \rightarrow \mathbf{B}$$

$$e \rightarrow e/\sqrt{\epsilon}$$

$$\phi \rightarrow \sqrt{\epsilon}\phi$$

$$\mathbf{A} \rightarrow \mathbf{A}$$

These equations may be solved in the same manner as before for the retarded and Lienard-Wiechert potentials.

- Cherenkov Radiation

- Radiation from relativistic charges moving in a plasma with $n_r \equiv \sqrt{\epsilon} > 1$.

  In this case, the velocity of the charges can exceed the phase velocity:

$$v_p = \frac{c}{n_r} < v < c \;\rightarrow\; \beta n_r > 1$$

  The beaming term of the Lienard-Wiechert potentials can vanish for an angle θ such that $\cos\theta = (n_r\beta)^{-1}$.

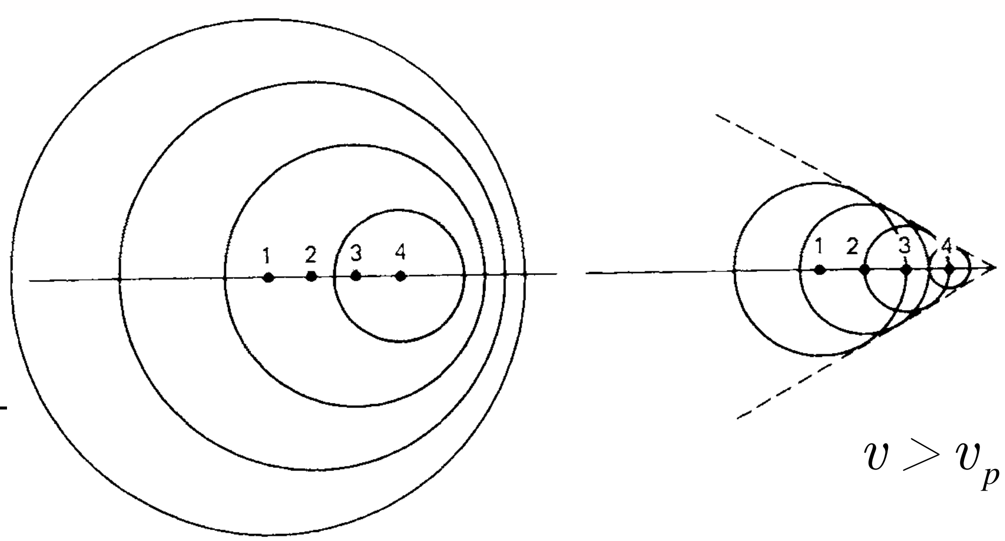
$$\kappa = 1 - (v/c)\cos\theta \rightarrow \kappa = 1 - \beta n_r \cos\theta$$

  The potentials become infinite at certain places. In consequence, the uniformly moving particle can now radiate.

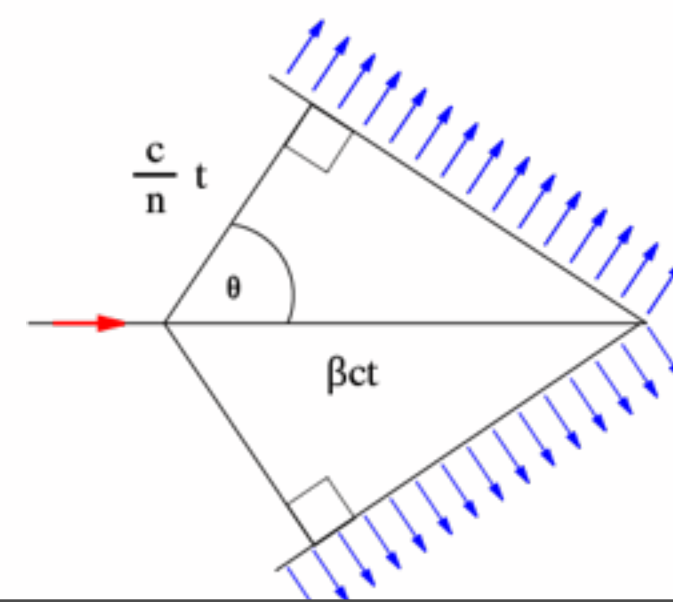
  Cherenkov cone: Outside the cone, points feel no potentials yet. Inside the cone, each point is intersected by two spheres. The resulting radiation is called Cherenkov radiation.

  A common analogy is the sonic boom of a supersonic aircraft or bullet.

$v < v_p = \frac{c}{n_r}$



$v > v_p$



- Razin-Tsytovich Effect

- When $n_r < 1$, Cherenkov radiation cannot occur.

  The critical angle defining the beaming effect in a vacuum was shown to be $\theta_b \sim 1/\gamma = \sqrt{1-\beta^2}$. But in a plasma we have instead

$$\theta_b \sim \sqrt{1-n_r^2\beta^2}$$

  If $n_r \ll 1$, and $\beta \sim 1$,

$$\theta_b \sim \sqrt{1-n_r^2} = \sqrt{1-\left(1-\frac{\omega_p^2}{\omega(\omega \pm \omega_B)}\right)} \approx \frac{\omega_p}{\omega}$$

  If $\omega < \gamma\omega_p$, $\theta_b > 1/\gamma$ and the beaming effect is suppressed.

  Below the frequency $\gamma\omega_p$, the synchrotron spectrum will be cut off because of the suppression of beaming. This is called the Razin-Tsytovich effect.

  As frequencies increase, θ_b decreases until it becomes of order of the vacuum value $1/\gamma$, and therefore the vacuum results apply.

  Therefore, the plasma medium effect is unimportant when $\omega \gg \gamma\,\omega_p$.