Radiative Processes in Astrophysics

Lecture 10
Nov 19 (Tue.), 2013
(last updated Nov. 19)

Kwang-Il Seon
UST / KASI

[Scattering from Electrons at Rest]

- Thomson Scattering

$$\epsilon = \epsilon_1$$

$$\frac{d\sigma_T}{d\Omega} = \frac{1}{2} r_0^2 \left(1 + \cos^2\theta\right)$$

$$\sigma_T = \frac{8\pi}{3} r_0^2$$

$\epsilon =$ energy of the incident photon

$\epsilon_1 =$ energy of the scattered photon

$$r_0 = \frac{e^2}{m_e c^2}$$

When $\epsilon = \epsilon_1$, the scattering is called coherent or elastic.

- **Compton scattering**: However, a photon carries momentum $\frac{h\nu}{c}$ and energy $h\nu$. Quantum effects appear in two ways.

(1) The scattering will no longer be elastic $(\epsilon \neq \epsilon_1)$ because of the recoil of the charge.

(2) The cross sections are altered by the quantum effects.

- Conservation of momentum and energy

Let the initial and final four-momenta of the photon: $\vec{P}_{\gamma i} = (\epsilon / c)(1, \mathbf{n}_i), \ \vec{P}_{\gamma f} = (\epsilon_1 / c)(1, \mathbf{n}_f)$

the initial and final momenta of the electron are: $\vec{P}_{ei} = (mc, \mathbf{0}), \ \vec{P}_{ef} = (E / c, \mathbf{p})$

Then, the conservation of momentum and energy is expressed by

$$\vec{P}_{ei} + \vec{P}_{\gamma i} = \vec{P}_{ef} + \vec{P}_{\gamma f}$$

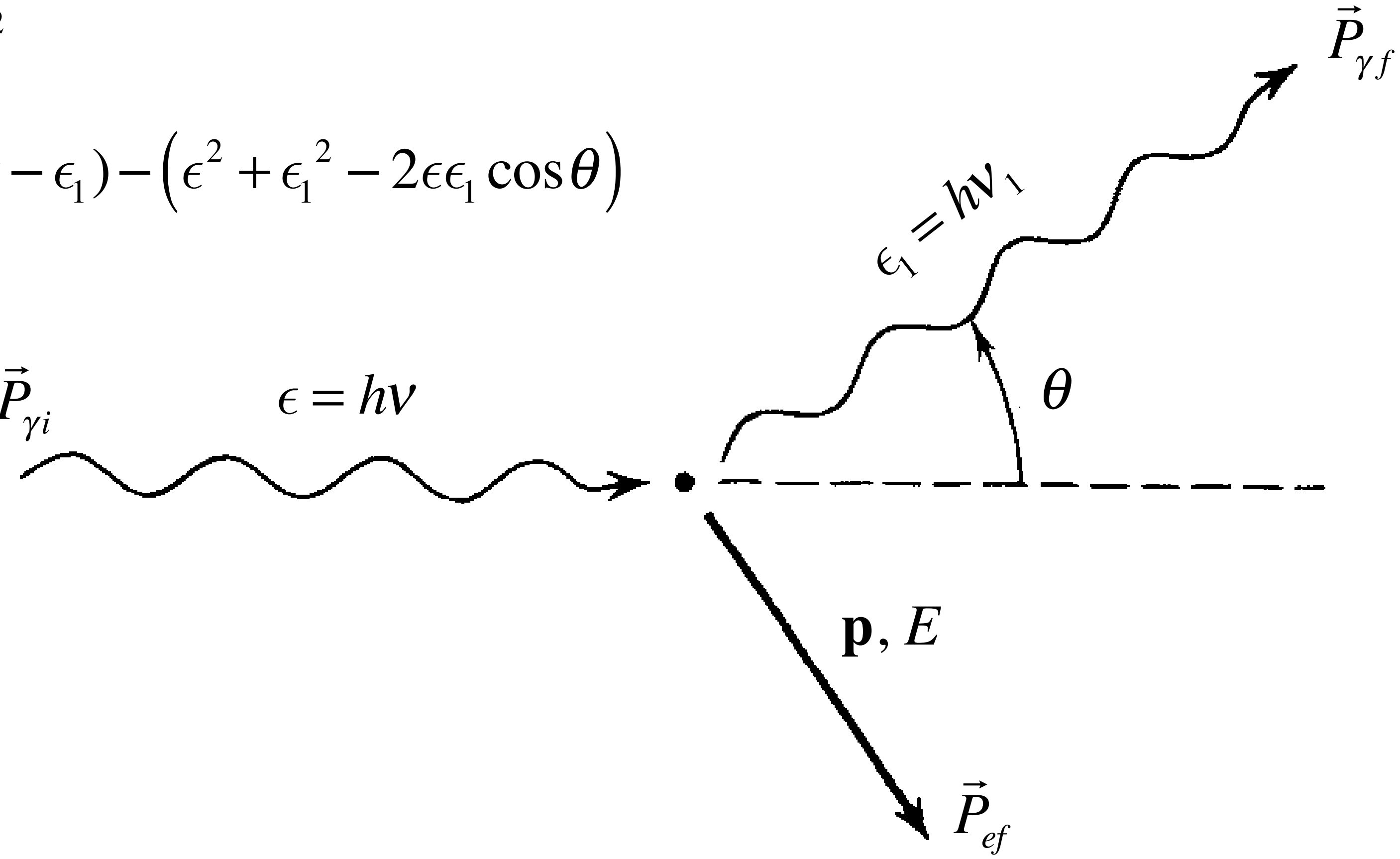
- Rearranging terms and squaring gives $\left|\vec{P}_{ef}\right|^2 = \left|\vec{P}_{ei} + \vec{P}_{\gamma i} - \vec{P}_{\gamma f}\right|^2$.

$$\left|\vec{P}_{ef}\right|^2 c^2 = \left|\vec{P}_{ei} + \vec{P}_{\gamma i} - \vec{P}_{\gamma f}\right|^2 c^2$$

$$E^2 - |\mathbf{p}|^2 c^2 = \left(mc^2 + \epsilon - \epsilon_1\right)^2 - \left|\epsilon \mathbf{n}_i - \epsilon_1 \mathbf{n}_f\right|^2$$

$$(mc^2)^2 = (mc^2)^2 + \epsilon^2 + \epsilon_1^2 - 2\epsilon\epsilon_1 + 2mc^2(\epsilon - \epsilon_1) - \left(\epsilon^2 + \epsilon_1{}^2 - 2\epsilon\epsilon_1 \cos\theta\right)$$

$$0 = mc^2\epsilon - \epsilon_1\left(\epsilon + mc^2 - \epsilon\cos\theta\right)$$



$$\therefore \epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1 - \cos\theta)}$$

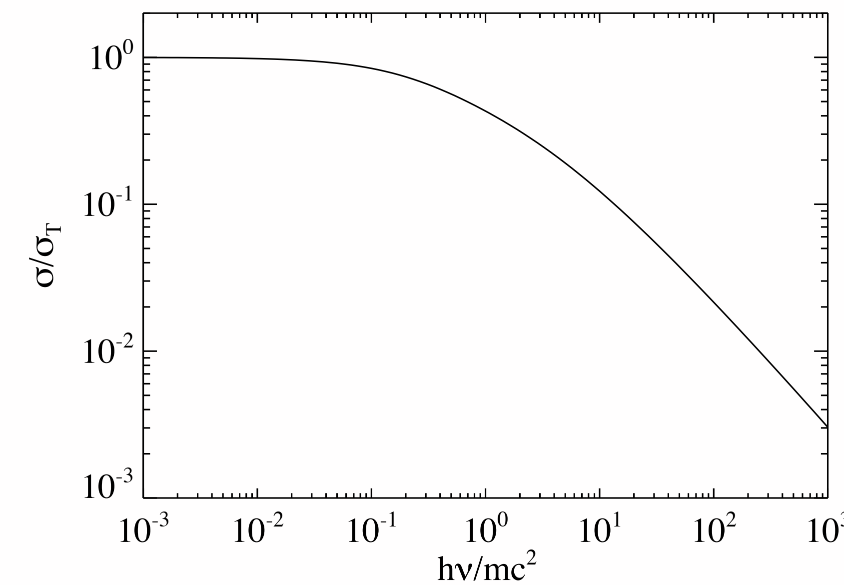
In terms of wavelength, $\lambda_1 - \lambda = \frac{h}{mc}(1 - \cos\theta)$

Compton wavelength: $\lambda_c \equiv \frac{h}{mc} = 0.02426 \mathring{A}$ for electrons

There is a wavelength change of the order of λ_c upon scattering.
For long wavelengths $\lambda \gg \lambda_c$ (i.e., $h\nu \ll mc^2$) the scattering is closely elastic.

- Klein-Nishina formula (the differential cross section for unpolarized radiation, QED)

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{16\pi}\frac{\epsilon_1^{\,2}}{\epsilon^2}\left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta\right)$$



- total cross section:

$$\sigma = 2\pi \int_{-1}^{1} \frac{d\sigma}{d\Omega} d\left(\cos\theta\right)$$

$$= \frac{3\sigma_T}{4}\left[\frac{1+x}{x^3}\left\{\frac{2x(1+x)}{1+2x} - \ln\left(1+2x\right)\right\} + \frac{1}{2x}\ln\left(1+2x\right) - \frac{1+3x}{\left(1+2x\right)^2}\right] \quad \text{where} \quad x \equiv \frac{h\nu}{mc^2}$$

(note $m_e c^2 = 511$ keV)

- Approximations:

nonrelativisitic regime: $\sigma \approx \sigma_T\left(1 - 2x + \frac{26x^2}{5} + \cdots\right), \quad x \ll 1$

extreme relativisitic regime: $\sigma \approx \frac{3}{8}\sigma_T \frac{1}{x}\left(\ln 2x + \frac{1}{2}\right), \quad x \gg 1$

[Scattering from Electrons in Motion]

- **Inverse Compton Scattering**: Whenever the moving electron has sufficient kinetic energy compared to the photon, net energy may be transferred from the electron to the photon.

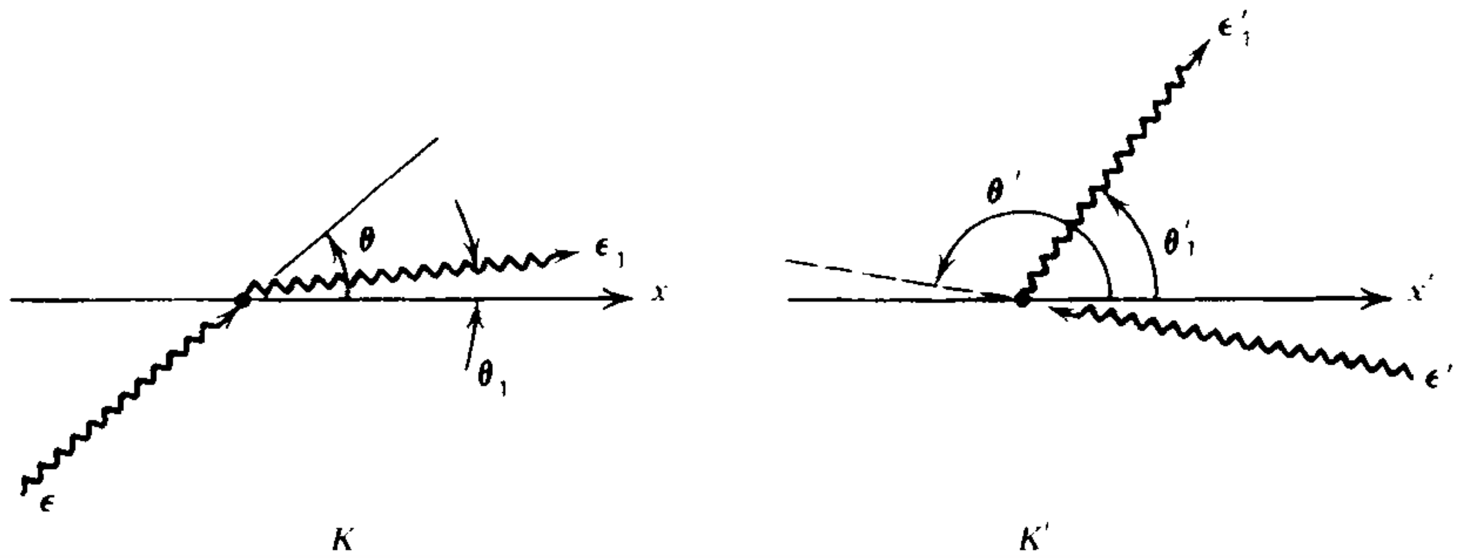
From Doppler shift formulas

$$\epsilon' = \epsilon\gamma(1-\beta\cos\theta)$$
$$\epsilon_1 = \epsilon_1'\gamma(1+\beta\cos\theta_1')$$

$$\epsilon_1' \approx \epsilon'\left[1-\frac{\epsilon'}{mc^2}(1-\cos\Theta')\right]$$

(if $\epsilon' \ll mc^2$)

$$\cos\Theta' = \cos\theta_1'\cos\theta' + \sin\theta'\sin\theta_1'\cos(\phi'-\phi_1')$$



In the case of relativistic electrons, $\gamma^2 - 1 \gg h\nu / mc^2$

$$\epsilon : \epsilon' : \epsilon_1' \approx 1 : \gamma : \gamma^2$$

providing that the condition for Thomson scattering in the rest frame is met $(\epsilon' \approx \epsilon\gamma \ll mc^2)$.

Therefore, the inverse Compton scattering converts a low-energy photon to a high-energy photon by a factor of order γ^2.

[Inverse Compton Power for Single Scattering]

- Assumptions:

  (1) isotropic distributions of photons and electrons.

  (2) The change in energy of the photon in the rest frame is negligible (Thomson scattering is applicable in the electron's rest frame). $\epsilon_1{}' \approx \epsilon'$

- Total power scattered in the electron's rest frame:

$$\frac{dE_1{}'}{dt'} = c\sigma_T \int \epsilon_1{}' n_\epsilon{}' d\epsilon'$$ where $n_\epsilon{}' d\epsilon'$ is the number density of incident photons.

- Recall

$$\frac{dE_1}{dt} = \frac{dE_1{}'}{dt'}$$ since energy and time transforms in the same way.

$$n_\epsilon d\epsilon = n_p d^3 p$$ where $n_p d^3 p$ is the number density of incident photons.

$d^3 p$ transforms in the same way as energy.

$$\therefore \frac{n_\epsilon d\epsilon}{\epsilon} = \frac{n_\epsilon{}' d\epsilon'}{\epsilon'}$$

- Thus we have the results

$$\frac{dE_1}{dt} = c\sigma_T \int \epsilon'^2 \frac{n_\epsilon' d\epsilon'}{\epsilon'} = c\sigma_T \int \epsilon'^2 \frac{n_\epsilon d\epsilon}{\epsilon} \qquad \epsilon' = \epsilon\gamma(1-\beta\cos\theta)$$

$$= c\sigma_T \gamma^2 \int (1-\beta\cos\theta)^2 \epsilon n_\epsilon \, d\epsilon$$

For an isotropic distribution of photons,

$$\left\langle (1-\beta\cos\theta)^2 \right\rangle = 1 + \frac{1}{3}\beta^2 \;\leftarrow\; \langle\cos\theta\rangle = 0, \left\langle \cos^2\theta \right\rangle = 1/3$$

we obtain $\dfrac{dE_1}{dt} = c\sigma_T \gamma^2 \left(1 + \dfrac{1}{3}\beta^2\right) U_{\text{ph}}$

$U_{\text{ph}} \equiv \int \epsilon n_\epsilon \, d\epsilon$ is the initial photon energy density.

Rate of decrease of the total initial photon energy is $\dfrac{dE_1}{dt} = -c\sigma_T U_{\text{ph}}$

Thus the net power lost bye the electron, and converted into increased radiation, is

$$P_{\text{compt}} \equiv \frac{dE_{\text{rad}}}{dt} = c\sigma_T U_{\text{ph}} \left[\gamma^2 \left(1 + \frac{1}{3}\beta^2\right) - 1 \right] = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_{\text{ph}} \quad \longleftarrow \quad \gamma^2 - 1 = \gamma^2\beta^2$$

- Recall that the formula for the synchrotron power emitted by each electron is

$$P_{\text{synch}} = \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_{\text{B}}$$

Therefore, $\frac{P_{\text{synch}}}{P_{\text{compt}}} = \frac{U_{\text{B}}}{U_{\text{ph}}}$

The radiation losses due to synchrotron emission and to inverse Compton effect are in the same ratio as the magnetic field energy density and photon energy density.

- Let $N(\gamma)d\gamma$ be the number of electrons per unit volume. Then, the total Compton power per unit volume is

$$P_{\text{tot}} = \int P_{\text{compt}} N(\gamma)\,d\gamma$$

(1) Power-law distribution of relativistic electrons $(\beta \sim 1)$

$$N(\gamma) = \begin{cases} C\gamma^{-p}, & \gamma_{\min} \le \gamma \le \gamma_{\max} \\ 0, & \text{otherwize} \end{cases} \longrightarrow P_{\text{tot}} = \frac{4}{3}\sigma_T c U_{\text{ph}} C(3-p)^{-1}\left(\gamma_{\min}^{3-p} - \gamma_{\max}^{3-p}\right)$$

(2) Thermal distribution of nonrelativistic electrons $(\gamma \sim 1)$

$$\left\langle \beta^2 \right\rangle = \left\langle v^2 / c^2 \right\rangle = 3kT / mc^2 \longrightarrow P_{\text{tot}} = \left(\frac{4kT}{mc^2}\right)\sigma_T c n_e U_{\text{ph}}$$

$\left(\frac{4kT}{mc^2}\right)$ → fractional photon energy gain

[Inverse Compton Spectra for Single Scattering]

- Approach: (1) Determine the spectrum for the scattering of photons of a single energy off electrons of a single energy, and then (2) Average over the actual distribution of photons and electrons.

- Assumptions:

  (1) Both the photons and electrons have isotropic distributions; the scattered photons are then also isotropically distributed.

  (2) Thomson scattering in the rest frame: $\gamma\epsilon_0 \ll mc^2, \;\; \epsilon_0' \approx \epsilon_1'$

  (3) Isotropic scattering in the rest frame: $\dfrac{d\sigma'}{d\Omega'} = \dfrac{1}{4\pi}\sigma_T$

  Even with these assumptions, we obtain the correct qualitative behavior of the results.

- We will use an intensity and emission coefficient based on photon number rather than energy.

$I(\epsilon)dAdtd\Omega d\epsilon$

= number of photons crossing are dA in time dt within solid angle $d\Omega$ and energy range $d\epsilon$

- Isotropic and monoenergetic photon field:

  in the observer frame, $I(\epsilon) = F_0\delta(\epsilon - \epsilon_0)$

  in the electron rest frame, $I'(\epsilon) = F_0\left(\dfrac{\epsilon'}{\epsilon}\right)^2 \delta(\epsilon - \epsilon_0)$ ⟵ $\dfrac{I}{\nu^2}$ = Lorentz invariant

From the Doppler formula $\epsilon = \epsilon' \gamma (1+\beta\mu')$, the incident intensity is

$$I'(\epsilon) = \left(\frac{\epsilon'}{\epsilon_0} \right)^2 F_0 \delta\left(\gamma\epsilon'(1+\beta\mu') - \epsilon_0 \right)$$

$$= \left(\frac{\epsilon'}{\epsilon_0} \right)^2 \frac{F_0}{\gamma\beta\epsilon'} \delta\left(\mu' - \frac{\epsilon_0 - \gamma\epsilon'}{\gamma\beta\epsilon'} \right)$$

Emission coefficient in the rest frame:

Let N = density of a electron beam

$$j'(\epsilon_1') = N'\sigma_T \frac{1}{4\pi} \int I'(\epsilon_1', \mu') d\mu'$$

$$= \frac{N'\sigma_T \epsilon_1' F_0}{2\epsilon_0^2 \gamma\beta}, \quad \text{if } \left| \frac{\epsilon_0 - \gamma\epsilon_1'}{\gamma\beta\epsilon_1'} \right| \le 1 \quad \text{or equivalently } \frac{\epsilon_0}{\gamma(1+\beta)} \le \epsilon_1' \le \frac{\epsilon_0}{\gamma(1-\beta)}$$

$$= 0, \qquad \text{otherwise.}$$

Emission coefficient in the observer's frame:

Recall that $\dfrac{j}{\epsilon}$ = Lorentz invariant

$$\epsilon_1' = \epsilon_1 \gamma (1 - \beta\mu_1)$$

$$N d^3\mathbf{x} = N' d^3\mathbf{x}', \quad d^3\mathbf{x} = \frac{d^3\mathbf{x}'}{\gamma}, \quad N = \gamma N'$$

$$j(\epsilon_1,\mu_1)=\frac{\epsilon_1}{\epsilon_1{}'}j'(\epsilon_1{}')$$

$$=\begin{cases}\dfrac{N'\sigma_T\epsilon_1F_0}{2\epsilon_0{}^2\gamma\beta}, & \text{if } \dfrac{\epsilon_0}{\gamma(1+\beta)}\le\epsilon_1{}'\le\dfrac{\epsilon_0}{\gamma(1-\beta)}\\ 0, & \text{otherwise.}\end{cases}$$

$$=\begin{cases}\dfrac{N\sigma_T\epsilon_1F_0}{2\epsilon_0{}^2\gamma^2\beta}, & \text{if } \dfrac{\epsilon_0}{\gamma^2(1+\beta)(1-\beta\mu_1)}\le\epsilon_1\le\dfrac{\epsilon_0}{\gamma^2(1-\beta)(1-\beta\mu_1)}\\ 0, & \text{otherwise.}\end{cases}$$

For an isotropic distribution of electrons $j(\epsilon_1)=\frac{1}{2}\int_{-1}^{+1}j(\epsilon_1,\mu_1)\,d\mu_1$

The integrand is nonzero only for a certain interval of μ_1:

$$\frac{\epsilon_0}{\gamma^2(1+\beta)(1-\beta\mu_1)}\le\epsilon_1\le\frac{\epsilon_0}{\gamma^2(1-\beta)(1-\beta\mu_1)}\rightarrow\frac{1}{\beta}\left[1-\frac{\epsilon_0}{\epsilon_1}(1+\beta)\right]\le\mu_1\le\frac{1}{\beta}\left[1-\frac{\epsilon_0}{\epsilon_1}(1-\beta)\right]$$

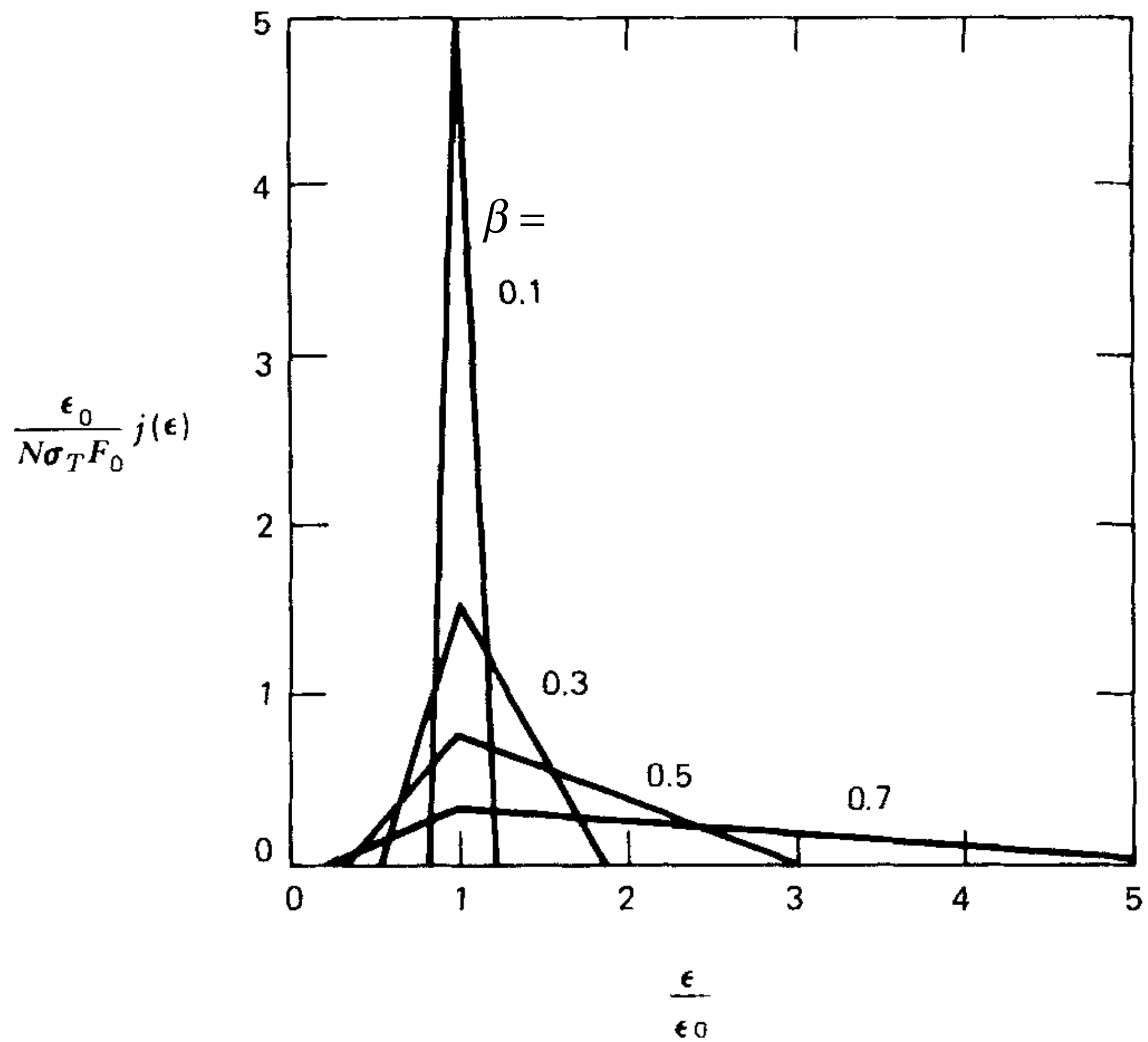
Since $-1\le\mu_1\le1$, the nonzero interval becomes:

$$-1\le\mu_1\le\frac{1}{\beta}\left[1-\frac{\epsilon_0}{\epsilon_1}(1-\beta)\right],\quad\text{for }\frac{1-\beta}{1+\beta}\le\frac{\epsilon_1}{\epsilon_0}\le1$$

$$\frac{1}{\beta}\left[1-\frac{\epsilon_0}{\epsilon_1}(1+\beta)\right]\le\mu_1\le1,\qquad\text{for }1\le\frac{\epsilon_1}{\epsilon_0}\le\frac{1+\beta}{1-\beta}$$

The result is:

$$j(\epsilon_1)=\frac{N\sigma_T F_0}{4\epsilon_0\gamma^2\beta^2}\begin{cases}(1+\beta)\frac{\epsilon_1}{\epsilon_0}-(1-\beta), & \frac{1-\beta}{1+\beta}\le\frac{\epsilon_1}{\epsilon_0}\le 1\\ (1+\beta)-\frac{\epsilon_1}{\epsilon_0}(1-\beta), & 1\le\frac{\epsilon_1}{\epsilon_0}\le\frac{1+\beta}{1-\beta}\\ 0, & \text{otherwise}\end{cases}$$



Note that

(1) For small β the curves are symmetrical about the initial photon energy.

(2) As β increases, the portion of the curve for

becomes more and more dominant.

$\int_0^\infty j(\epsilon_1)d\epsilon_1 = N\sigma_T F_0$: the conservation of number of photons upon scattering

$\int_0^\infty j(\epsilon_1)(\epsilon_1-\epsilon_0)d\epsilon_1 = N\sigma_T \frac{4}{3}\gamma^2\beta^2\epsilon_0 F_0$: the average increase in photon energy per scattering

For extreme relativistic case $(\beta \approx 1, \gamma \gg 1)$

$$j(\epsilon_1) = \frac{N\sigma_T F_0}{4\epsilon_0 \gamma^2 \beta^2}\left[(1+\beta) - \frac{\epsilon_1}{\epsilon_0}(1-\beta)\right], \quad 1 \le \frac{\epsilon_1}{\epsilon_0} \le \frac{1+\beta}{1-\beta}$$

$$= \frac{N\sigma_T F_0}{4\epsilon_0 \gamma^2}\frac{1+\beta}{\beta^2}\left(1 - \frac{\epsilon_1}{\epsilon_0}\frac{1-\beta}{1+\beta}\right)$$

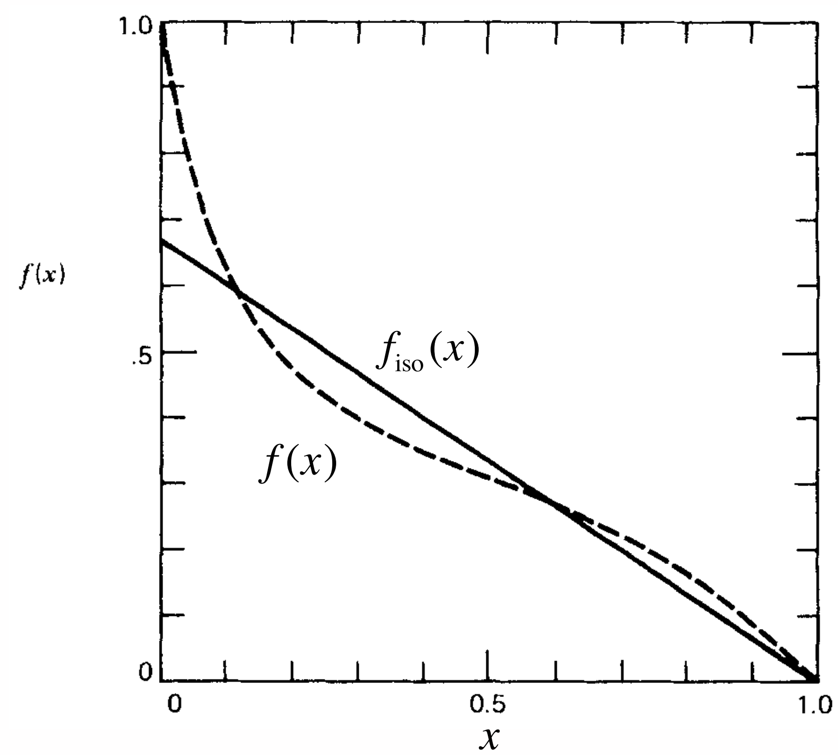
$$\approx \frac{N\sigma_T F_0}{4\epsilon_0 \gamma^2} 2\left(1 - \frac{\epsilon_1}{\epsilon_0}\frac{1}{4\gamma^2}\right)$$

Note: $1 \le \frac{\epsilon_1}{\epsilon_0} \le \frac{1+\beta}{1-\beta} \rightarrow 1 \le \frac{\epsilon_1}{\epsilon_0} \lesssim 4\gamma^2 \rightarrow 0 \lesssim x \lesssim 1$

$$j(\epsilon_1) \approx \frac{3N\sigma_T F_0}{4\epsilon_0 \gamma^2} f_{\rm iso}(x) \quad \text{where} \quad x \equiv \frac{\epsilon_1}{4\gamma^2 \epsilon_0}, \quad f_{\rm iso} \equiv \begin{cases} \frac{2}{3}(1-x), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

When the exact angular dependence in the differential cross section is included,

$$f(x) = 2x\ln x + x + 1 - 2x^2, \ \ 0 < x < 1$$



- Power law distribution of relativistic electrons:

$N(\gamma) = C\gamma^{-p}$: electron distribution

$n_\epsilon = \dfrac{4\pi I(\epsilon)}{c}$: photon number density

Total scattered power per volume per energy is

$$\frac{dE}{dVdtd\epsilon_1} = 4\pi\epsilon_1 j(\epsilon_1)$$

$$= \frac{3c\sigma_T}{4}\int d\epsilon\left(\frac{\epsilon_1}{\epsilon}\right)n_\epsilon \int_{\gamma_1}^{\gamma_2} d\gamma\left(C\gamma^{-p-2}\right)f(x)$$

$$= 3\sigma_T c C 2^{p-2}\epsilon_1^{-(p-1)/2}\int d\epsilon \epsilon^{(p-1)/2} n_\epsilon \int_{x_1}^{x_2} dx x^{(p-1)/2} f(x)$$

Suppose that $\gamma_2 \gg \gamma_1$ and that n_ϵ peaks at some value $\bar{\epsilon}$.

Then $x_1 \equiv \epsilon_1 / (4\gamma_1^2\epsilon) \to 0$ and the second integral is independent of ϵ_1.

$x_2 \equiv \epsilon_2 / (4\gamma_2^2\epsilon) \to \infty$

The spectral index is to be identical to the case of synchrotron emission.

$$\frac{dE}{dVdtd\epsilon_1} \propto \epsilon_1^{-(p-1)/2}$$

[Repeated Scattering: The Compton y Parameter]

- We restrict our considerations to situations in which the Thomson limit applies: $\gamma\epsilon \ll mc^2$
- <u>**Compton *y* parameter**</u>, to determine whether a photon will significantly change its energy in traversing the medium:

$$y \equiv \begin{pmatrix} \text{average fractional} \\ \text{energy change per} \\ \text{scattering} \end{pmatrix} \times \begin{pmatrix} \text{mean number of} \\ \text{scatterings} \end{pmatrix}$$

When $y \gtrsim 1$, the total photon energy and spectrum will be significantly altered; whereas for $y \ll 1$, the total energy is not much changed.

- <u>**Average fractional energy change per scattering**</u> (for a thermal distribution of electrons)

<u>Consider first the nonrelativistic limit</u>.

$$\epsilon_1{}' \approx \epsilon'\left[1 - \frac{\epsilon'}{mc^2}(1-\cos\Theta)\right] \rightarrow \frac{\Delta\epsilon'}{\epsilon'} \equiv \frac{\epsilon_1{}' - \epsilon'}{\epsilon'} = -\frac{\epsilon'}{mc^2} \qquad \text{: angle average}$$

In the lab frame to lowest order, this must be of the form

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

To calculate α, image that the photons and electrons are in complete equilibrium but interact only through scattering.

Assume that the photon density is sufficiently small that stimulated processes can be neglected. Then, we obtain the Wien's law for the photon distribution:

$$n_\epsilon = K\epsilon^2 \exp\left(-\frac{\epsilon}{kT}\right)$$

We have the averages

$$\langle\epsilon\rangle \equiv \int \epsilon n_\epsilon \, d\epsilon \Big/ \int n_\epsilon \, d\epsilon = 3kT$$

$$\langle\epsilon^2\rangle \equiv \int \epsilon^2 n_\epsilon \, d\epsilon \Big/ \int n_\epsilon \, d\epsilon = 12(kT)^2$$

For this case, no net energy can be transferred from photons to electrons, so

$$\Delta\epsilon = 0 = -\frac{\langle\epsilon^2\rangle}{mc^2} + \frac{\alpha kT}{mc^2}\langle\epsilon\rangle = \frac{3kT}{mc^2}(\alpha - 4)kT \quad \rightarrow \quad \alpha = 4$$

Thus for nonrelativisitic electrons in thermal equilibrium, the energy transfer per scattering is

$$\boxed{(\Delta\epsilon)_{\mathrm{NR}} = \frac{\epsilon}{mc^2}(4kT - \epsilon)}$$

Note that if electrons have high enough temperature relative to incident photons, the photons gain energy. This is the inverse Compton scattering.

If $\epsilon > 4kT$, on the other hand, energy is transferred from photons to electrons.

- In the ultrarelativistic limit $(\gamma \gg 1)$, ignoring the energy transfer in the electron rest frame,

$$\frac{P_{\rm compt}}{dE_1/dt} = \frac{(4/3)\sigma_T c\gamma^2\beta^2 U_{\rm ph}}{\sigma_T c U_{\rm ph}} = \frac{4}{3}\gamma^2\beta^2 \;\rightarrow\; (\Delta\epsilon)_{\rm R} \approx \frac{4}{3}\gamma^2\epsilon$$

For a thermal distribution of ultrarelativistic electrons,

$$\langle\gamma^2\rangle = \frac{\langle\epsilon^2\rangle}{(mc^2)^2} = 12\left(\frac{kT}{mc^2}\right)^2 \;\rightarrow\; \boxed{(\Delta\epsilon)_{\rm R} \approx 16\epsilon\left(\frac{kT}{mc^2}\right)^2}$$

- **Mean number of scatterings**,

Recall that, for a pure scattering medium,

$$\begin{pmatrix}\text{mean number of}\\ \text{scatterings}\end{pmatrix} \approx {\rm Max}\left(\tau_{\rm es}, \tau_{\rm es}^2\right) \qquad \text{where} \quad \tau_{\rm es} \sim \rho\kappa_{\rm es}R$$

$$\kappa_{\rm es} = \frac{\sigma_T}{m_p} = 0.40\ {\rm cm}^2\ g^{-1} \quad \text{for ionized hydrogen}$$

$$R = \text{size of the finite medium}$$

- **Compton *y* parameter**:

$$\boxed{y_{\rm NR} = \frac{4kT}{mc^2}{\rm Max}(\tau_{\rm es}, \tau_{\rm es}^2) \qquad y_{\rm R} = 16\epsilon\left(\frac{kT}{mc^2}\right)^2 {\rm Max}(\tau_{\rm es}, \tau_{\rm es}^2)}$$

[Repeated Scattering: Spectra and Power]

- A power-law spectrum may be a natural consequence of a power-law distribution of electrons.
- We will show that a power-law photon distribution can also be produced from repeated scattering off a nonpower-law electron distribution.

Let A = the mean amplification of photon energy per scattering

$$A \equiv \frac{\epsilon_1}{\epsilon}$$

$$\sim \frac{4}{3}\langle \gamma^2 \rangle = 16\left(\frac{kT}{mc^2}\right)^2 \quad \text{for thermal electron distribution}$$

mean photon energy = ϵ_i

intensity = $I(\epsilon_i)$ at ϵ_i

After k scattering, the photon energy will be $\epsilon_k \sim \epsilon_i A^k$.

For a optically thin scattering medium ($\tau_{\rm es} < 1$), the probability of a photon undergoing k scattering before escaping the medium is $p_k(\tau_{\rm es}) \sim \tau_{\rm es}{}^k$.

The emergent intensity at energy ϵ_k is given by

$$I(\epsilon_k) \sim I(\epsilon_i)\tau_{\rm es}^k \sim I(\epsilon_i)\tau_{\rm es}^{\ln(\epsilon_k/\epsilon_i)/\ln A} = I(\epsilon_i)\left(\frac{\epsilon_k}{\epsilon_i}\right)^{\ln\tau/\ln A}$$

$$\therefore I(\epsilon_k) \sim I(\epsilon_i)\left(\frac{\epsilon_k}{\epsilon_i}\right)^{-\alpha} \quad \text{where} \quad \alpha \equiv \frac{-\ln\tau_{\rm es}}{\ln A} \longrightarrow \text{power-law shape}$$

- Total Compton power in the output spectrum is given by

$$P \propto \int I(\epsilon_k)\, d\epsilon_k = I(\epsilon_i)\epsilon_i \left[\int x^{-\alpha}\, dx \right]$$

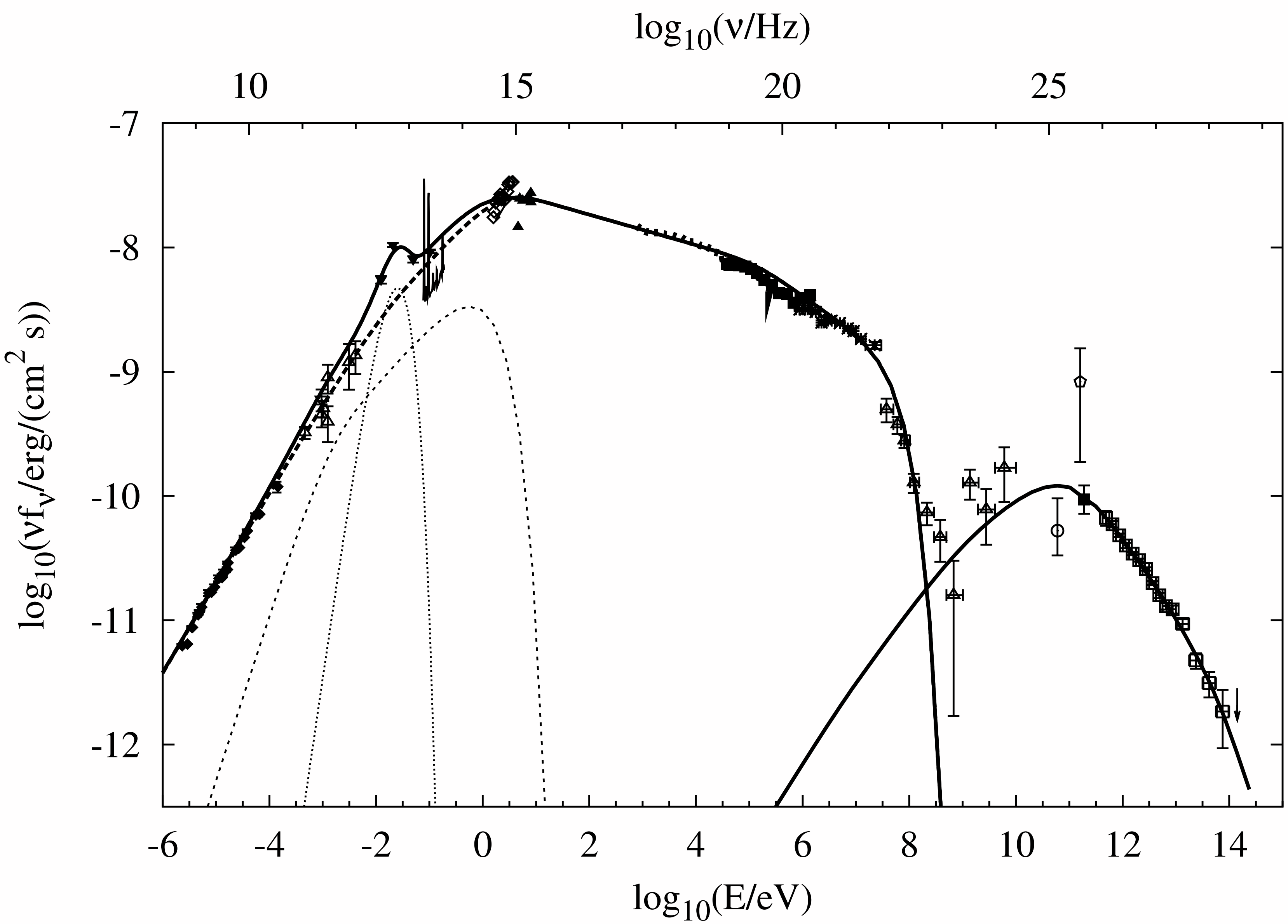
The factor in square brackets is approximately the factor by which the initial power $I(\epsilon_i)\epsilon_i$ is amplified in energy.

Clearly, this amplification will be important if $\alpha \ll 1$. Therefore, energy amplification of a soft photon input spectrum is important when

$$\frac{-\ln \tau_{\rm es}}{\ln A} \leq 1 \;\rightarrow\; \ln(\tau_{\rm es} A) \geq 1 \;\rightarrow\; y = A\tau_{\rm es} \gtrsim 1$$

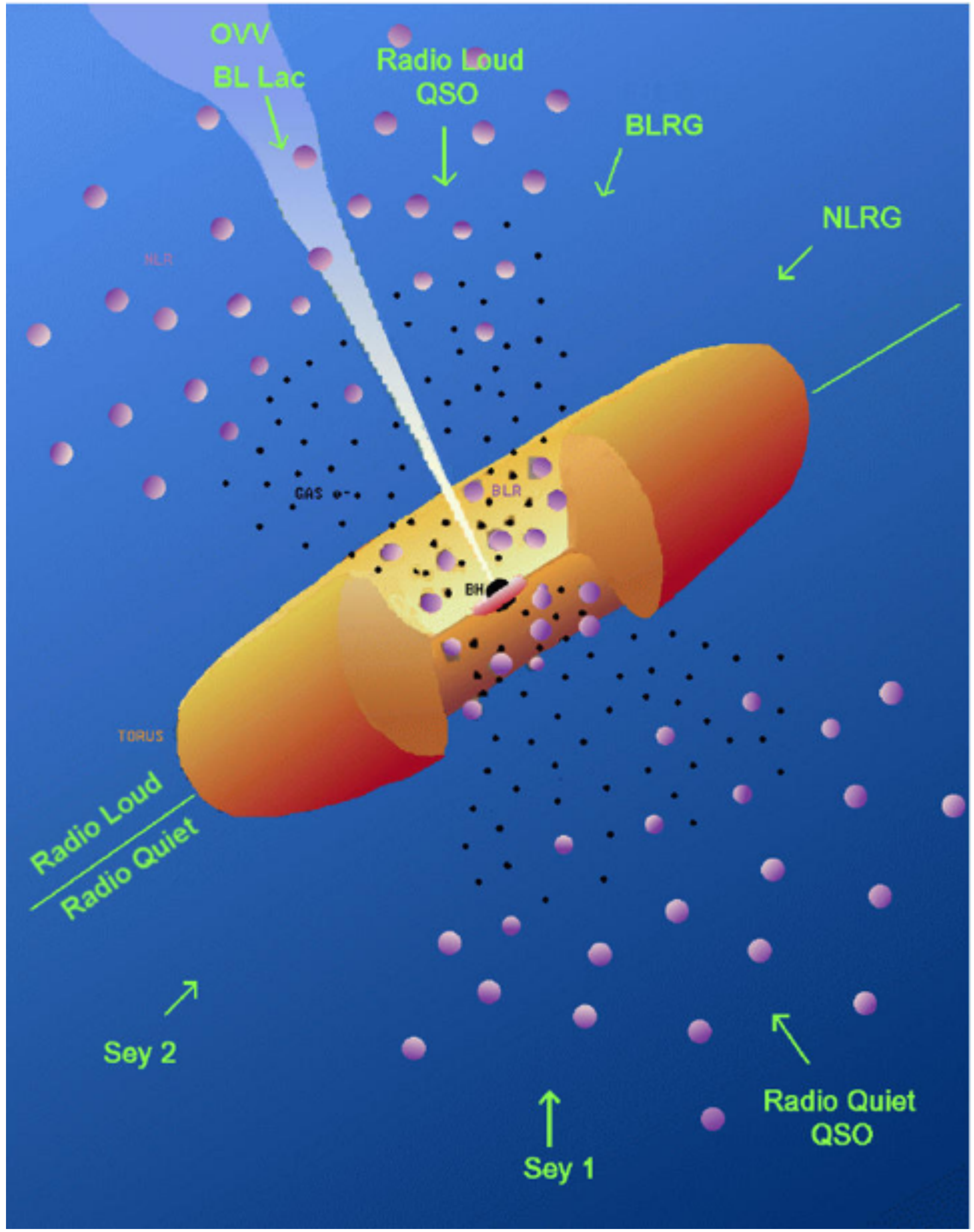
[Synchrotron self-Compton (SSC) emission]

- The modification of the photon spectrum by Compton scattering is called **Comptonization**.
- Relativistic electrons in the presence of a magnetic field will surely emit synchrotron radiation at some level. The photons will undergo inverse compton scattering by the very same electrons that emitted them in the first place. Such scattering must take place before the synchrotron photon leaves the source region. This is the **synchrotron self-Compton (SSC) process**.
- **Crab nebula**



Active Galactic Nuclei

- A Unified Model for AGN



- **Blazars**: If the observer view is more or less normal to the accretion disk, the action close to the core becomes visible. The observer considered to lie within the jet beam. Such objects are known as blazars or as BL Lacertae objects.
- Blazars have SEDs that are typically two peaked. The peak at lower frequency is attributed to synchrotron radiation and the one at higher frequency to IC scattering.

  The lower-energy case (LBL blazar) extends from the radio to the gamma-ray bands but is quiet in the TeV band. The higher-energy case (HBL blazar) reaches TeV energies but is quiet in the radio range.

LBL: Low-frequency peaked BL Lacs
HBL: High-frequency peaked BL Lacs